On the unreasonable effectiveness of post-Newtonian theory in gravitational physics

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Washington University, St. Louis

Chandrasekhar Centennial Symposium, 15 - 16 October 2010
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The unreasonable effectiveness of mathematics in the natural sciences

By Eugene Wigner

Communications in Pure and Applied Mathematics

“....the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and ... there is no rational explanation for it.”
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- Introduction
- The problem of motion & radiation - a history
- Post-Newtonian theory
- "Unreasonable effectiveness"
  - PPN framework
  - Binary pulsars
  - Gravitational-wave kicks
  - PN-numerical waveform matching
- Chandra: some personal remarks

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The post-Newtonian approximation

\[ \varepsilon \sim \left( \frac{v}{c} \right)^2 \sim \left( \frac{Gm}{rc^2} \right) \sim \left( \frac{p}{\rho c^2} \right) \]

\[ g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon h^{(1)}_{\mu\nu} + \varepsilon^2 h^{(2)}_{\mu\nu} + \ldots \]

\[ G_{\mu\nu} = 8\pi T_{\mu\nu} \quad (G = c = 1) \]

\[ T_{\mu\nu} = \rho u_\mu u_\nu + p(u_\mu u_\nu + g_{\mu\nu}) \]
The problem of motion & radiation

- Geodesic motion
- 1916 - Einstein - gravitational radiation (wrong by factor 2)
- 1916 - Droste & De Sitter - n-body equations of motion
- 1918 - Lense & Thirring - motion in field of spinning body
- 1937-38 - center-of-mass acceleration (Levi-Civita/Eddington-Clark)
- 1938 - EIH paper
- 1960s - Fock & Chandrasekhar - PN approximation
- 1967 - the Nordtvedt effect
- 1974 - numerical relativity - BH head-on collision
- 1974 - discovery of PSR 1913+16
- 1976 - Ehlers et al - critique of foundations of EOM
- 1976 - PN corrections to gravitational waves (EWW)
- 1979 - measurement of damping of binary pulsar orbit
- 1990s to now - EOM and gravitational waves to HIGH PN order
  - driven by requirements for GW detectors
  - $(v/c)^{12}$ beyond Newtonian gravity
A Global Network of Interferometers

- LIGO Hanford 4.2 km
- LIGO Livingston 4 km
- GEO Hannover 600 m
- TAMA Tokyo 300 m
- Virgo Cascina 3 km
LISA: a space interferometer for 2020
**Inspiralling Compact Binaries - The Workhorse Source**

- Fate of the binary pulsar in 100 My
- GW energy loss drives pair toward merger

**LIGO-VIRGO**
- Last few minutes (10K cycles) for NS-NS
- 40 - 700 per year by 2014
- BH inspirals could be more numerous

**LISA**
- MBH pairs($10^5 - 10^7 M_\odot$) in galaxies to $Z \sim 15$
- EMRIs

**Extremely accurate theoretical templates needed to maximize detection and parameter estimation**
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Einstein's Equations

\[ G_{\mu\nu} = 8\pi T_{\mu\nu} \]

"Relaxed" Einstein's Equations

\[ h^{\mu\nu} \equiv \eta^{\mu\nu} - \sqrt{-g} g^{\mu\nu} \]

\[ \partial_v h^{\mu\nu} = 0 \]

\[ h^{\mu\nu} = -16\pi (-g)(T^{\mu\nu} + t^{\mu\nu}) \]

\[ h^{\mu\nu} = 4 \int_{C} \frac{\tau^{\mu\nu} (t-|x-x'|,x')}{|x-x'|} \, d^3 x' \]

\[ \nabla_v T^{\mu\nu} = 0, \text{ or } \partial_v \tau^{\mu\nu} = 0 \]
PN equations of motion for compact binaries

\[ \vec{a} = -\frac{\dot{m}}{r^3} \vec{x} + 1PN + 1PN_{SO} + 1PN_{SS} + 2PN + 2.5PN \]

+ 3PN  
+ 3.5PN  
+ 3.5PN_{SO}  
+ 3.5PN_{SS}

B = Blanchet, Damour, Iyer et al
F = Futamase, Itoh
S = Schäfer, Jaranowski
W = WUGRAV
Gravitational energy flux for compact binaries

\[ \dot{E} = \dot{E}_{\text{quad}} + 1PN \quad \text{Wagoner & CW 76} \]

\[ \dot{E} + 1PN_{SO} + 1PN_{SS} \quad \text{W} \]

\[ \mathcal{L} = \frac{32c^5}{5G} x^2 x^5 \left\{ 1 + \left( -\frac{1247}{336} - \frac{35}{12} \right) \nu x + 4\pi x^{3/2} + \left( -\frac{44711}{9072} + \frac{9271}{504} \nu + \frac{65}{18} \nu^2 \right) x^2 \right. \]

\[ + \left( -\frac{8191}{672} - \frac{583}{24} \right) \pi x^{5/2} \]

\[ + \left[ \frac{6643739519}{69854400} + \frac{16}{3} \pi^2 - \frac{1712}{105} \nu - \frac{856}{105} \ln(16x) \right. \]

\[ + \left( -\frac{134543}{7776} + \frac{41}{48} \pi^2 \right) \nu - \frac{94403}{3024} \nu^2 - \frac{775}{324} \nu^3 \]

\[ + \left( -\frac{16285}{504} + \frac{214745}{1728} \nu + \frac{193385}{3024} \nu^2 \right) \pi x^{7/2} + O \left( \frac{1}{c^8} \right) \} \]

\[ + 3.5PN \quad \text{B} \]
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### Bounds on the PPN Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Effect or Experiment</th>
<th>Bound</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Time delay</td>
<td>$2.3 \times 10^{-5}$</td>
<td>Cassini tracking</td>
</tr>
<tr>
<td></td>
<td>Light deflection</td>
<td>$4 \times 10^{-4}$</td>
<td>VLBI</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Perihelion shift</td>
<td>$3 \times 10^{-3}$</td>
<td>$J_2 = 2 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>Nordtvedt effect</td>
<td>$2.3 \times 10^{-4}$</td>
<td>LLR, $\eta &lt; 3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Earth tides</td>
<td>$10^{-3}$</td>
<td>gravimeters</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Orbit polarization</td>
<td>$10^{-4}$</td>
<td>LLR</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2 \times 10^{-4}$</td>
<td>J2317+1439</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Spin precession</td>
<td>$4 \times 10^{-7}$</td>
<td>Sun axis</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>Self-acceleration</td>
<td>$4 \times 10^{-20}$</td>
<td>Pulsar spindown</td>
</tr>
<tr>
<td>$\zeta_1$</td>
<td>--</td>
<td>$2 \times 10^{-2}$</td>
<td>Combined bounds</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>Binary acceleration</td>
<td>$4 \times 10^{-5}$</td>
<td>PSR 1913+16</td>
</tr>
<tr>
<td>$\zeta_3$</td>
<td>Newton's 3rd law</td>
<td>$10^{-8}$</td>
<td>Lunar acceleration</td>
</tr>
<tr>
<td>$\zeta_4$</td>
<td>--</td>
<td></td>
<td>Not independent</td>
</tr>
</tbody>
</table>

\[ \eta = 4\beta - \gamma - 3\cdot10\zeta/3 - \alpha_1 + 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3 \]

Bound on scalar-tensor gravity: $\omega > 40,000$
The Binary Pulsar: Is strong gravity "effaced"?

Discovery: 1974
Pulse period: 59 ms (16cps)
Orbit period: 8 hours

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Keplerian</strong></td>
<td></td>
</tr>
<tr>
<td>Pulse Period (ms)</td>
<td>59.029997929613(7)</td>
</tr>
<tr>
<td>Orbit Period (days)</td>
<td>0.322997448930(4)</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.6171338(4)</td>
</tr>
<tr>
<td><strong>Post-Keplerian</strong></td>
<td></td>
</tr>
<tr>
<td>Periastron Shift (dω/dt °/yr)</td>
<td>4.226595(5)</td>
</tr>
<tr>
<td>Pulsar Clock Shifts (ms)</td>
<td>4.2919(8)</td>
</tr>
<tr>
<td>Orbit Decay (dP_b/dt 10^{-12})</td>
<td>-2.4184(9)</td>
</tr>
</tbody>
</table>
PSR 1913+16: Concordance with GR

\[ m_p = 1.4411(7) \, M_{\text{sun}} \quad m_c = 1.3874(7) \, M_{\text{sun}} \]
The Double Pulsar

J 0737-3039
- 0.10 day orbit
- two pulsars seen!
- $\frac{d\omega}{dt} = 17^\circ/yr$
- $\sin i = 0.9995$
- $\frac{dP_b}{dt} \sim 6\%$
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Numerical Relativity meets PN Theory
Radiation of momentum and the recoil of massive black holes

General Relativity
- Interference between quadrupole and higher moments
  Peres (62), Bonnor & Rotenberg (61), Papapetrou (61), Thorne (80)
- “Newtonian effect” for binaries
  Fitchett (83), Fitchett & Detweiler (84)
- 1 PN correction term
  Wiseman (92)

Astrophysics
- MBH formation by mergers is affected if BH ejected from early galaxies
  Schnittman (07)
- Ejection from dwarf galaxies or globular clusters
- Displacement from center could affect galactic core
  Merritt, Milosavljevic, Favata, Hughes & Holz (04)
  Favata, Hughes & Holz (04)

J0927+2943 - 2600 km/s? Komossa et al 2008
How black holes get their kicks
Favata, et al

Getting a kick out of numerical relativity
Centrella, et al

Total recoil: the maximum kick....
Gonzalez et al

A swift kick in the astrophysical compact object
Boot, Foot, et al
Radiation of momentum to 2PN order: ISCO & Plunge

- Calculate relevant multipole moments to 2PN order: quadrupole, octupole, current quadrupole, etc
- Calculate momentum flux for quasi-circular orbit \([x=(m\omega)^{2/3} \approx (v/c)^2]\)
  \[\text{recoil} = -\text{flux}\]
  \[
  \frac{d\vec{P}}{dt} = - \frac{464}{105} \frac{\delta m}{m} \eta^2 x^{11/2} \left[ 1 + \left( - \frac{452}{87} - \frac{1139}{522} \eta \right) x + \frac{309}{58} \pi x^{3/2} \right]
  + \left( - \frac{71345}{22968} + \frac{36761}{2088} \eta + \frac{147101}{68904} \eta^2 \right) x^2 \hat{\lambda}
  \]

- Integrate up to ISCO (6m) for adiabatic inspiral
- Match quasicircular orbit at ISCO to plunge orbit in Schwarzschild
- Integrate with respect to “proper \(\omega\)” to horizon (\(x \to 0\))

Blanchet, Qusailah & CW (2005)
Comparing 2PN kick with numerical relativity

Baker et al (GSFC), gr-qc/0603204
Close-limit approximation at 2PN order

- 2PN two-body metric in harmonic coordinates
- linear in $G$
- transform to Schwarzschild coordinates
- expand in powers of $r_{12}/r$
- $g_{\mu\nu} = g_{\mu\nu}^{(S)} + h_{\mu\nu}^{(e)} + h_{\mu\nu}^{(o)}$
- go to RW gauge and find $\Psi_{l,m}^{(o)}$, $\Psi_{l,m}^{(e)}$
- use as initial data for the wave equation
  $$\frac{\partial^2}{\partial t^2} \Psi_{l,m}^{(e,o)} - \frac{\partial^2}{\partial r_*^2} \Psi_{l,m}^{(e,o)} + V_{\ell}^{(e,o)} \Psi_{l,m}^{(e,o)} = 0$$
- find fluxes of $P$, $E$, $J$

Le Tiec & Blanchet, CQG 27 045008 (2010); arXiv:0910.4593
Net kick from 2PN plunge & ringdown

- 2PN Plunge only
- 2PN Plunge + ringdown
- NR
E and J radiated from PN plunge & CL ringdown

3PN inspiral E, J flux combined with 2PN close limit approximation
NRm3PN
Comparing High-order PN with Numerical Waveforms

Baker et al. gr-qc/0612024
NRm3PN
Comparing High-order PN with Numerical Waveforms

Hannam et al. arXiv:0706.1305
Boyle et al. arXiv:0710.0158

\[ r \approx 4.6M \]
Reasonable vs. Unreasonable

Computing the merger of black-hole binaries: the IBBH problem

Patrick R. Brady, Jolien D. E. Creighton, and Kip S. Thorne
Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125
(22 April 1998)

Gravitational radiation arising from the inspiral and merger of binary black holes (BBH's) is a promising candidate for detection by kilometer-scale interferometric gravitational wave observatories. This paper discusses a serious obstacle to searches for such radiation and to the interpretation of any observed waves: the inability of current computational techniques to evolve a BBH through its last ~ 10 orbits of inspiral (~ 100 radians of gravitational-wave phase). A new set of numerical-relativity techniques is proposed for solving this "Intermediate Binary Black Hole" (IBBH) problem: (i) numerical evolutions performed in coordinates co-rotating with the BBH, in which the metric coefficients evolve on the long timescale of inspiral, and (ii) techniques for mathematically freezing out gravitational degrees of freedom that are not excited by the waves.
NRm3PN
Comparing High-order PN with Numerical Waveforms

Boyle et al.
gr-qc/0804.4124

$r \approx 3.7 M$
A picnic with Chandra and Laletha, 1973
Chandra at WU 1995
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