Graphical Causal Models for Time Series Econometrics: Some Recent Developments and Applications

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10 December 2009

MAX-PLANCK-GESELLSCHAFT

Mini Symposium: Causality and Time Series Analysis
NIPS 2009, Vancouver

Scope

- Application of methods of causal search to the problem of finding the appropriate causal order for the Structural Vector Autoregressive models (SVAR).

Overview

- VAR and SVAR model
- Causal search methods: graphical models
- Application to the linear/Gaussian setting
- Extensions:
  - Nonparametric setting
  - Non-Gaussian case: application of a method based on ICA

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- ▷ Extensions:
  - Nonparametric setting
  - Non-Gaussian case: application of a method based on ICA

VAR vs. SVAR model

Basic VAR model (reduced-form):

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + u_t. \quad (1)$$

- Y_t: $(y_{t1}, \ldots, y_{tk})'$;
- A_j $(j = 1, \ldots, p)$ are $k \times k$ coefficient matrices;
- u_t is the vector white noise process;
- $E(u_t u_t') = \Sigma_u$.

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Wold representation (in case of stationarity):

$$Y_t = \sum_{j=0}^{\infty} \Phi_j u_{t-j}, \tag{2}$$

where $\Phi_j = \sum_{i=1}^{j} \Phi_{j-i} A_i$

But for any nonsingular $k \times k$ matrix P we get:

$$Y_t = \sum_{j=0}^{\infty} \Phi_j P P^{-1} u_{t-j} = \sum_{j=0}^{\infty} \Psi_j \varepsilon_{t-j}, \tag{3}$$

where $\varepsilon_{t-j} = P^{-1} u_{t-j}$ and $\Psi_j = \Phi_j P$ $(j = 0, 1, 2, ...)$.

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If we premultiply equation (1) by P^{-1} we get

$$P^{-1}Y_t = P^{-1}A_1 Y_{t-1} + \ldots + P^{-1}A_p Y_{t-p} + P^{-1}u_t. \tag{4}$$

SVAR model (structural form):

$$\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \ldots + \Gamma_p Y_{t-p} + \varepsilon_t, \tag{5}$$

where $\Gamma_0 = P^{-1}, \Gamma_j = P^{-1}A_j$ $(j = 1, \ldots, p)$.

$\triangleright$ choice of P (Γ_0) based on information about the contemporaneous causal structure.

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VAR vs. SVAR model

- choice of P (Γ_0) in the literature:
  - *Choleski* decomposition such that: P is lower diagonal and $\Omega = E(\varepsilon_t \varepsilon_t') = I_k$.
  - a priori (theoretical, institutional) zero-restrictions;
- Our proposal: inferring P (Γ_0) starting from the estimated residuals $\hat{u}_t$
  - conditional independence relations $\longrightarrow$ causal relationships (graphical models)
  - independent component analysis (in case of non-Gaussianity)

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Graphical models

Graphs have two functions:

- Representation of causal structures
- Representation of causal independence relations

Graphical models

Representation of causal structures:

▷ edge: causal influence

- Undirected edges: $X — Y$ (ambiguous causal influence between X and Y)
- Directed edges: $X \longrightarrow Y$, $X \longleftarrow Y$
- Bi-directed edges: $X \longleftrightarrow Y$

▷ DAGs: Directed acyclic graphs (only directed edges).

Graphical models

Representation of conditional independence relations:

▷ edge: statistical dependence

- $X \perp\!\!\!\perp Y \mid \emptyset$: no edge between X and Y
- $X \not\!\perp\!\!\!\perp Y$: $X - Y$; $X \longrightarrow Y$; $X \longleftarrow Y$
- $X \perp\!\!\!\perp Z \mid Y$: $X \longrightarrow Y \longrightarrow Z$; $X \longleftarrow Y \longleftarrow Z$; $X \longleftarrow Y \longrightarrow Z$
- $X \not\!\perp\!\!\!\perp Z \mid Y$: $X \longrightarrow Y \longleftarrow Z$

Graphical models

Rules of Inference:

▷ edge: from conditional independence $\mapsto$ causal relationships

- DAGs among $V_1, \ldots, V_k$
  - Causal Markov Condition: *conditioned on its parents every node is independent of its nondescendants*; or: *conditioned on its direct causes every variable is independent of its non-effects*
  - Faithfulness Condition: *every conditional independence relation among* $V_1, \ldots, V_k$ *is entailed by the Causal Markov Condition*

Search algorithm:

PC, SGS, modified PC algorithm (Spirtes, Glymour and Scheines 2000):

- Input: conditional independence tests
- Output: set of Markov equivalent DAGs

▷ Start: complete undirected graph among $V_1, \ldots, V_k$

▷ First step: elimination of edges whenever $\perp\!\!\!\perp$

▷ Second step: statistical orientation of edges

- search for *unshielded colliders*: $X \longrightarrow Y \longleftarrow Z$

▷ Third step: logical orientation of edges

- $X \longrightarrow Y — Z \quad \mapsto \quad X \longrightarrow Y \longrightarrow Z$
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GMs applied to SVAR

- Cfr. Swanson and Granger (1997), Bessler and Lee (2002), Demiralp and Hoover (2003) (among others)
- Estimate reduced form VAR

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + u_t. \tag{6}$$

- King-Stock-Plosser-Watson (1991) updated data set
  US data 1947:2 - 1994:1 (quarterly data)

$$Y = \begin{pmatrix} C \\ I \\ M \\ Y \\ R \\ \Delta P \end{pmatrix} \begin{matrix} \text{per capita consumption} \\ \text{per capita investment} \\ \text{money M2 / price} \\ \text{per capita private income} \\ \text{nominal interest rate} \\ \text{price inflation} \end{matrix}$$

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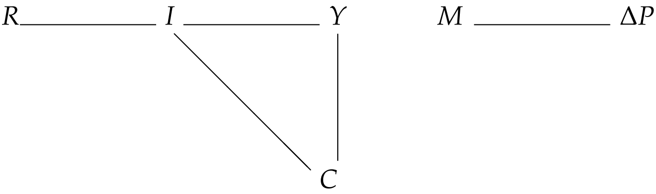
- Causal graph among $u_{1t}, \ldots, u_{kt}$ $\equiv$ causal graph among $y_{1t}, \ldots, y_{kt}$
- C.I. relations among $u_{1t}, \ldots, u_{kt}$ tested via Wald tests on zero-partial correlations
- Gaussianity assumption

Results (Moneta 2008):



Configurations $R \longrightarrow I \longleftarrow Y$ and $R \longrightarrow I \longleftarrow C$ are excluded.

Possibilities:

- sensitivity analysis
- bootstrap analysis (cfr. Demiralp, Hoover and Perez 2008)

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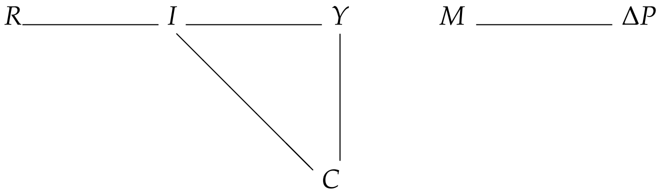

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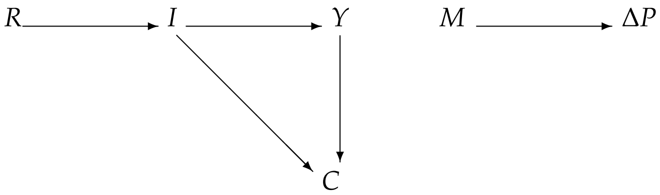
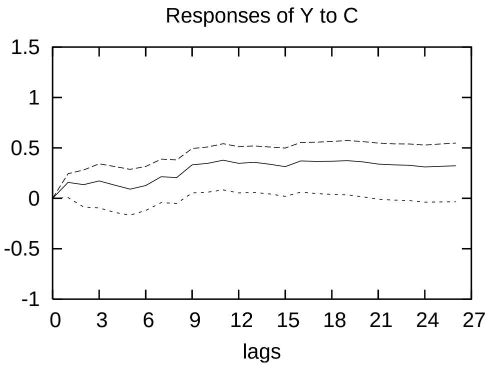


Impulse Response Analysis



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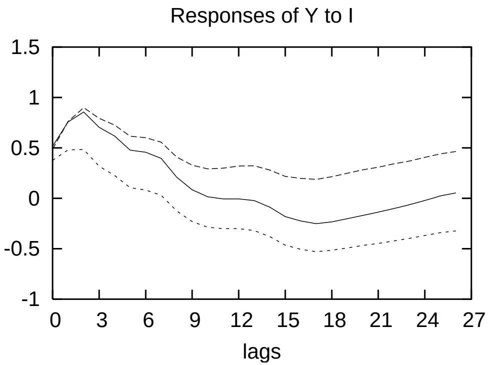

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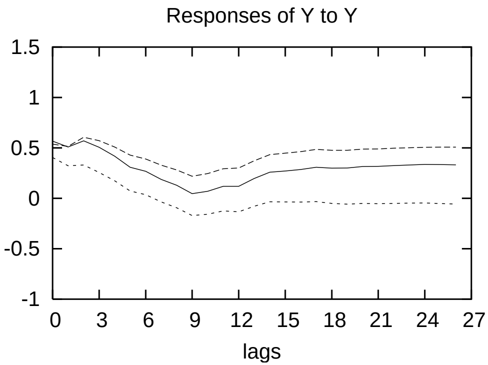

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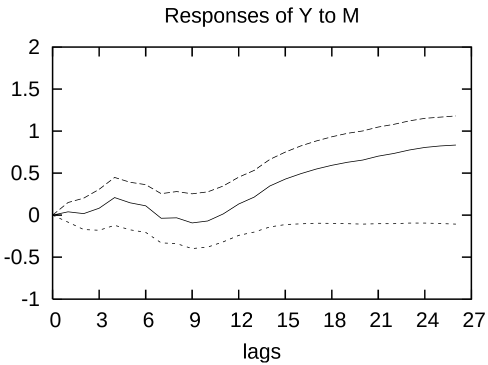

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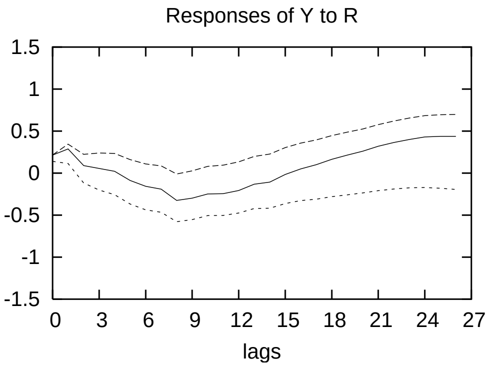

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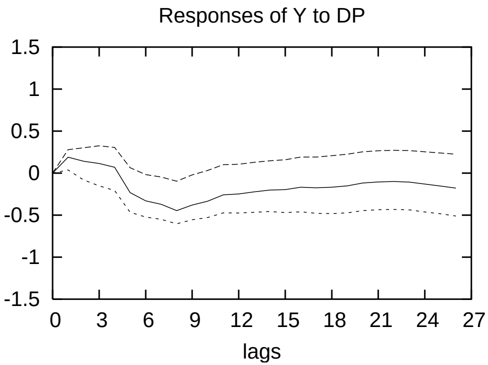

Extensions

- Non-parametric approach
  - Start from non-parametric tests of conditional independence
- Semi-parametric approach
  - Assumption of linearity and non-Gaussianity

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Nonparametric approach

Chlaß and Moneta (2009):

- $X \perp\!\!\!\perp Y \mid Z$ iff $f(X|Y,Z) = f(X|Z)$
- Test $\hat{f}(X,Y,Z)\hat{f}(Z) = \hat{f}(X,Z)\hat{f}(Y,Z)$.
- Distance measures between kernel density functions:
  - Euclidean distance (Szekely and Rizzo 2004; Baringhaus and Franz 2004)
  - Weighted Hellinger distance (Su and White 2008)
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Moneta, Entner, Hoyer and Coad (2009)

▷ VAR-LiNGAM algorithm (Hyvärinen, Shimizu and Hoyer 2008)

- Estimate the reduced-form VAR
- Check that the residuals are non-Gaussian
- Use an ICA algorithm to decompose the residuals matrix
- Order the variables (residuals) so as to obtain a matrix P that is close to lower triangular
- Once the instantaneous effects are identified, identify lagged effects
- No need of assumptions such as Faithfulness.

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Empirical Application

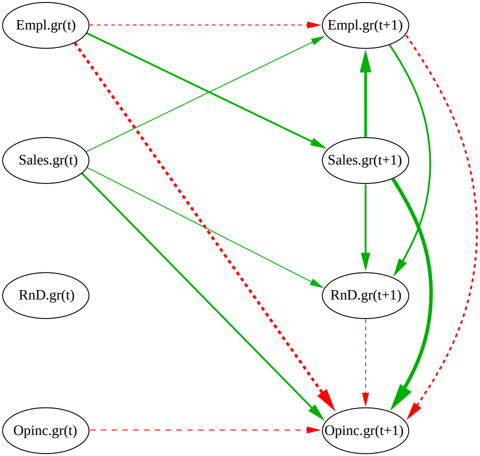
Panel VAR on Firm Growth and R&D Expenditures

Coad-Rao (2007) data set: US firm 1973:2004 (manufacturing sector).

- Employees (growth rate)
- Total sales (growth rate)
- R&D expenditure (growth rate)
- Profits (growth rate)

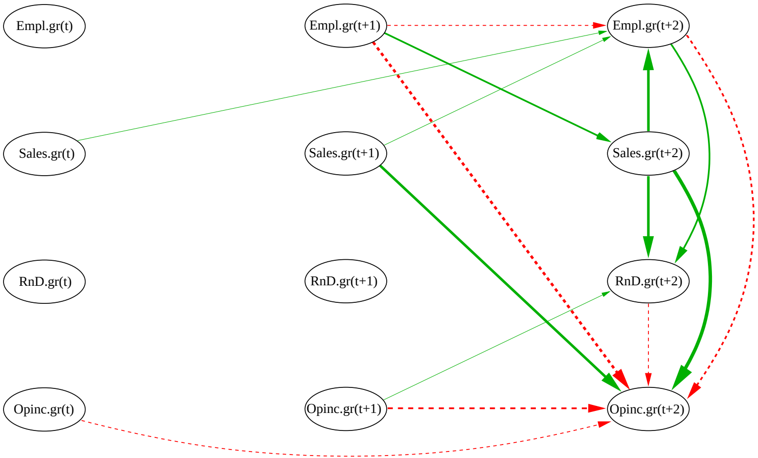
LAD (least absolute deviation) estimation

Structural coefficients one-lag model:



Solid green arrows: positive causal influence; dashed red arrows: negative influence. Only major effects are displayed.

Structural coefficients two-lags model:



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Conclusions

- ▷ Identification of causal structure in time series / non-experimental settings
- ▷ SVAR model: possibility of applying causal search to i.i.d. residuals
- ▷ Graphical causal search applied to linear / Gaussian data
- ▷ Assumptions (rules of inference): Causal Markov and Faithfulness Condition
- ▷ Extensions:
  - Nonparametric conditional independence tests. No distributional assumptions but same rules of inference.
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