Learning Nonlinear Dynamic Models

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Posterior update:
\[
P(y_{t+1}|X_{1:t}) \propto \sum_{y_t} P(y_t|X_{1:t-1})P(x_t|y_t)P(y_{t+1}|y_t).
\]

Prediction of future events:
\[
P(x_{t+1}|X_{1:t}) = \sum_{y_{t+1}} P(x_{t+1}|y_{t+1})P(y_{t+1}|X_{1:t}).
\]
Computing the posterior $P(y_{t+1}|X_{1:t})$ is difficult.

- Linearize nonlinear function: Extended Kalman Filter.
- Use approximations, e.g., particle filtering.

Inputs are high-dimensional and highly-structured.
• Posterior $P(y_{t+1}|X_{1:t})$ is approximated by a family of distributions parameterized by $u_{t+1} \in U$:

$$P(y_{t+1}|X_{1:t}) \approx P(y_{t+1}|u_{t+1}).$$

• $u_{t+1}$ is a sufficient statistic for the posterior $P(y_{t+1}|X_{1:t})$.
• $u_{t+1}$ is a deterministic parameter.
Sufficient Posterior Representation (SPR):

\[ P(x_{t+1}|X_{1:t}) \approx P(x_{t+1}|u_{t+1}). \]

• Posterior update: \( u_{t+1} = B(x_t, u_t) \).

Give an arbitrary value to the initial state \( u_1 \):

\[ u_2 = A(x_1) = B(x_1, u_1) \].
**Sufficient Posterior Representation**

- **Prediction:**
  \[ p(x_{t+1} | X_{1:t}) = C(u_{t+1}). \]

**Key Observation:** \( A, B, \) and \( C \) are deterministic.
A sufficient posterior representation of a dynamic model (SPR-DM) is given by:

- **Observed sequence:** \( \{x_t\} \)
- **Unobserved hidden “state”:** \( \{u_t\} \)
- **State initialization map:** \( u_2 = A(x_1) \)
- **State update map:** \( u_{t+1} = B(x_t, u_t) \)
- **Prediction map:** \( p(x_{t+1}|X_{1:t}) = C(u_{t+1}) \).
Learning SPR-DM

First prediction problem:
\[ p(x_2|x_1) = C(A(x_1)) . \]

- State is “that information which summarizes the first observation in predicting the second observation”.
- Internal state can come from any learning algorithm.
Learning SPR-DM State Evolution

Second prediction problem:
\[ p(x_t | X_{1:t-1}) = C(B(x_{t-1}, u_{t-1})). \]

- A state and an observation is used to predict the next state, reusing the state prediction from previous step.
Learning SPR-DM

Pretraining: Local learning.

Fine-tuning: Backpropagation through time.
Invertibility of SPR-DM

• To show consistency, we need the notion of invertibility.

   The SPR-DM is invertible if there exist a function $R$ such that for all $t$, $R(C(u_t)) = u_t$.

• Let $C = p(X_{t:t+k} | u_t)$.

Invertibility: if $u_t$ and $u'_t$ induce the same short range behavior $X_{t:t+k}$, then they are identical –

They induce the same behavior for all $X_{t:\infty}$. 

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– Learning Nonlinear Dynamic Models –
• $u_2 = A(x_1) = \sigma (A^\top x_1 + b)$
• $u_t = B(x_{t-1}, u_{t-1}) = \sigma (B_1^\top x_{t-1} + B_2^\top u_{t-1} + b)$
• $\hat{x}_t = C(u_t) = C^\top u_t + a$

where $\sigma(y) = 1/(1 + \exp(-y))$. 

– Learning Nonlinear Dynamic Models –
Motion Capture Data

- Sequences of 3D joint angles plus body orientation and translation
- Various walking styles: normal, drunk, graceful, gangly, chicken, etc.
- 30 training and 8 test sequences, each of length 50.
- Each time step was represented by a vector of 58 real-valued numbers.
• Comparison: 20-dimensional nonlinear model, 20 and 100-state HMM’s, and simple linear models (conditioned on 2 and 5 previous time steps).
Weizmann Video Data

- Video sequences of nine human subjects.
- Various actions: waving one hand, waving two hands, jumping, and bending.
- 36 training and 10 test sequences, each of length 50.
- Each time step was represented by a vector of 464 real-valued numbers.
Comparison: 50-dimensional nonlinear model, 50 and 100-state HMM’s, and linear models.
Thank you.