Sparse Log Gaussian Processes via MCMC for Spatial Epidemiology

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Spatial epidemiology

- Mortality data from Statistics Finland

- Large scale:
  - over million deaths with various reasons
  - 30 years (one month accuracy)
  - in quarter million locations (250m accuracy)

- Complex
  - spatial effects
  - temporal effects
  - covariates
The model

- The mortality is modeled as a Poisson process with mean $E \mu$

  $Y \sim \text{Poisson}(E \mu)$,

  where $E$ is the standardised expected number of deaths

- $\log(\mu)$ is given a GP prior with zero mean

  $\log(\mu) = f(x_i, x_j) \sim \mathcal{GP}(0, k(x_i, x_j))$
A fully independent training conditional (FITC) (Snelson & Ghahramani, 2006; Quinonero-Candela & Rasmussen, 2005) sparse approximation is used to speed up GP computations.
Approximate conditional posterior of latent values

- Following Christensen et al (2006), approximate posterior precision is obtained as

\[ \Sigma^{-1} = K^{-1} + \Sigma_1^{-1}, \]

where

\[ \Sigma_1^{-1} \approx -\frac{\partial^2 \log(\text{Poisson}(E\lambda))}{\partial f^2} = E\mu, \]

where \( \mu \) is approximated with its prior mean 1.
Transformation when $K$ is reduced rank

- Using matrix inversion lemma and eigen decomposition following equations for transformation are obtained

\[
USU^T = \hat{\Lambda}^{1/2} \Lambda^{-1} K_{f,u} \left( K_{u,u} + K_{u,f} \Lambda^{-1} K_{f,u} \right)^{-1} K_{u,f} \Lambda^{-1} \hat{\Lambda}^{1/2} \ast
\]

\[
f = \hat{\Lambda}^{1/2} (\tilde{f} + UD^{-1} U^T \tilde{f} - UU^T \tilde{f})
\]

\[
\tilde{f} = \hat{\Lambda}^{-1/2} f + UDU^T \hat{\Lambda}^{-1/2} f - UU^T \hat{\Lambda}^{-1/2} f,
\]

where $U$ and $S$ are matrices of eigenvectors and eigenvalues of the right hand side of the $\ast$ respectively. $D_{ii} = \sqrt{1 - S_{ii}}$ and $\hat{\Lambda} = \left( \Sigma_l^{-1} + \Lambda^{-1} \right)^{-1}$. 
Example 1
Example 2

Full GP

Sparse GP

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