Causality estimation from time series in the presence of NOISE

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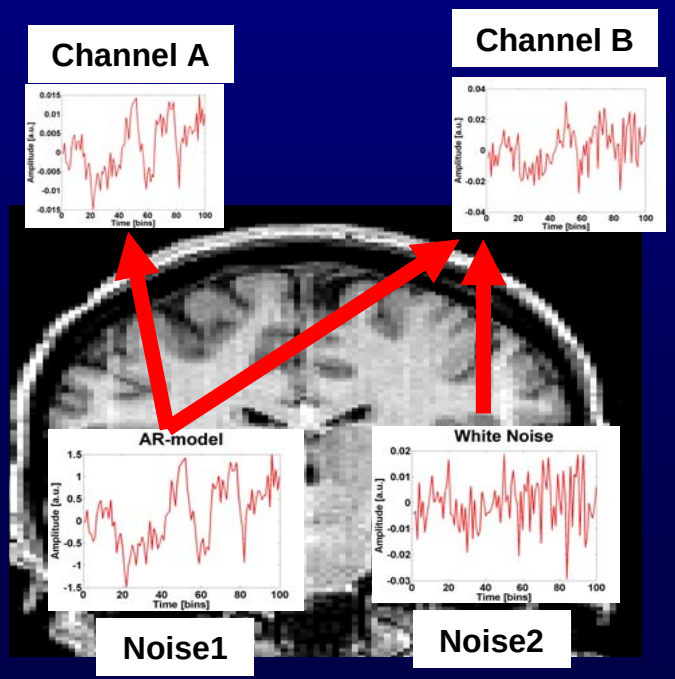
Motivation

- Many measurements like EEG/MEG/fMRI are extremely noisy

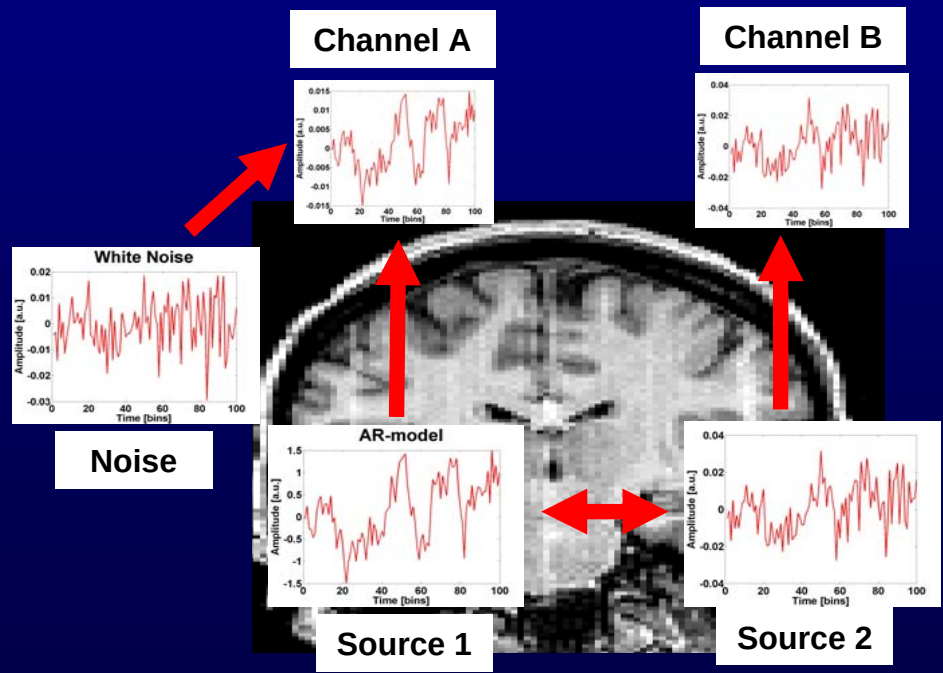
Mixtures of independent sources:

Do we estimate fake direction?

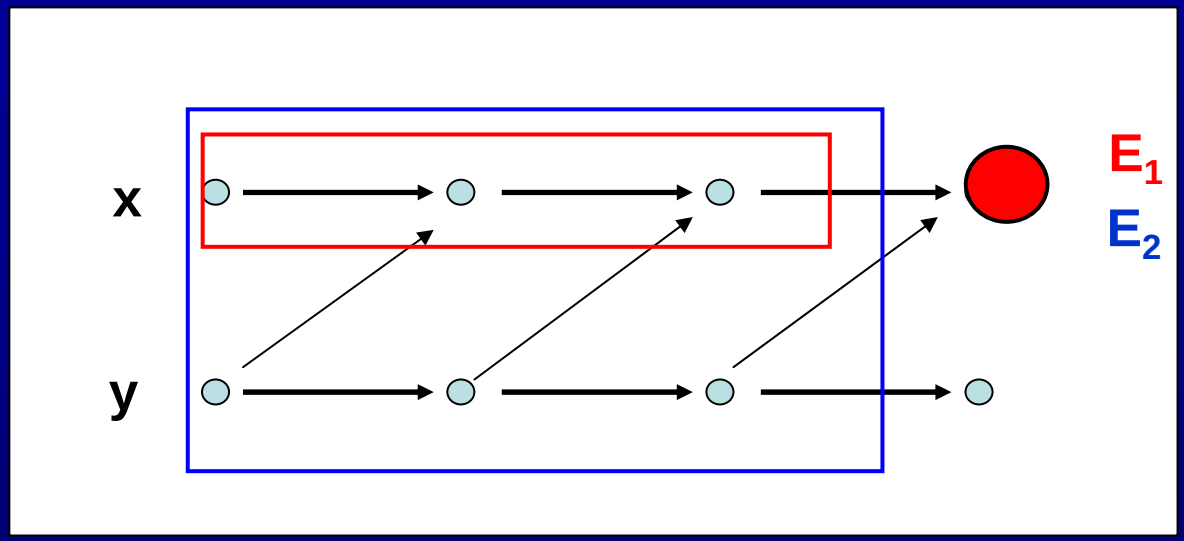


Additive noise:

Do we estimate wrong direction?



Granger causality



$$F_{y\to x} = \log\left(\frac{E_1}{E_2}\right)$$

$$\hat{G} = F_{x\to y} - F_{y\to x}$$

$$G = \frac{\hat{G}}{std(\hat{G})}$$

Phase Slope Index

C(f) **(complex) coherence between two sensors**

$$\widehat{\Psi} = \mathrm{Im} \sum_{f} C^{*}(f) C(f + \delta f)$$

$$\Psi = \frac{\widehat{\Psi}}{std(\widehat{\Psi})}$$

- **'average' phase slope**
- **vanishes for mixtures of independent sources**

Nolte, et.al., Phys Rev Let., 2008

Simulated Challenge Data: signal + mixed noise

Signal $x_i(t)$:
unidirectional AR-Model

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \sum_{p=1}^{10} \begin{pmatrix} A_{11}(p) & 0 \\ A_{21}(p) & A_{22}(p) \end{pmatrix} \begin{pmatrix} x_1(t-p) \\ x_2(t-p) \end{pmatrix} + \begin{pmatrix} \xi_1(t) \\ \xi_2(t) \end{pmatrix}$$

$A_{ij}(p)$ random, direction random, ξ_i uniform

Noise $y_i(t)$: :
3 independent sources with random spectrum

$$y_i(t) = \sum_{p=1}^{10} \tilde{A}_{ii}(p) y_i(t-p) + \eta_i(t)$$

$\tilde{A}_{ii}(p)$ random, η_i uniform

Data $z_i(t)$: :
Random superposition of signal and mixed noise

$$\vec{z}(t) = (1-\gamma)\frac{\vec{x}(t)}{\|X\|} + \gamma \frac{B\vec{y}(t)}{\|BY\|}$$

B random 2×3 matrix, γ random $\in [0,1]$

Simulated Challenge Data

- 1000 examples, 6000 time points, 2 sensors
- Task: estimate direction for all 1000 examples
- Matlab code to generate examples is provided
- Main idea: class of problems rather than specific problems

Form of solutions:

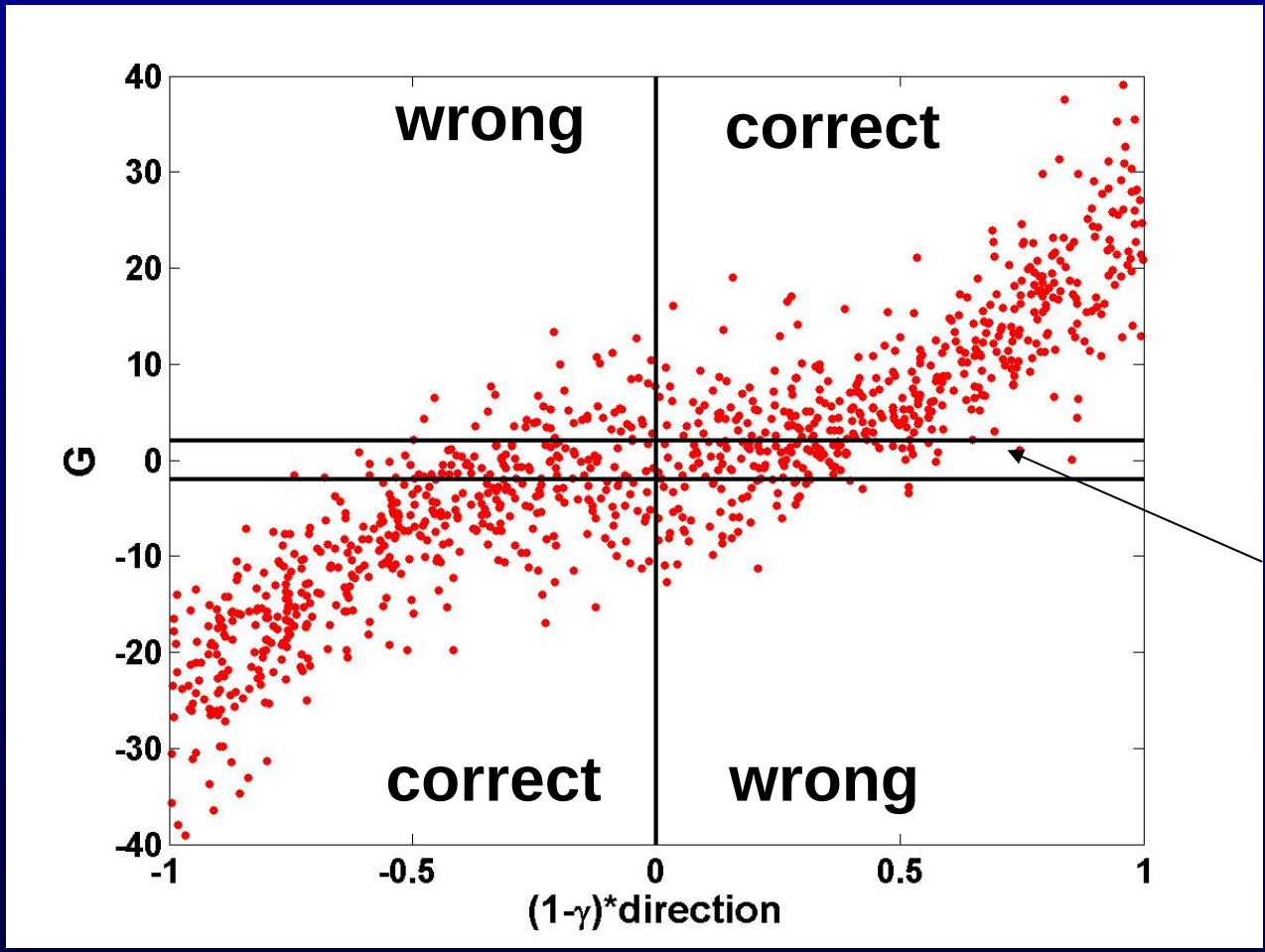
- True solutions: “x to y” or “y to x”
- Possible answers: “x to y” or “y to x” or “I don’t know”

How it is counted:

- you get +1 for each correct, -10 for each wrong, 0 for each “I don’t know”
- Main Idea: Evaluate evidence !!

Results for Granger Causality

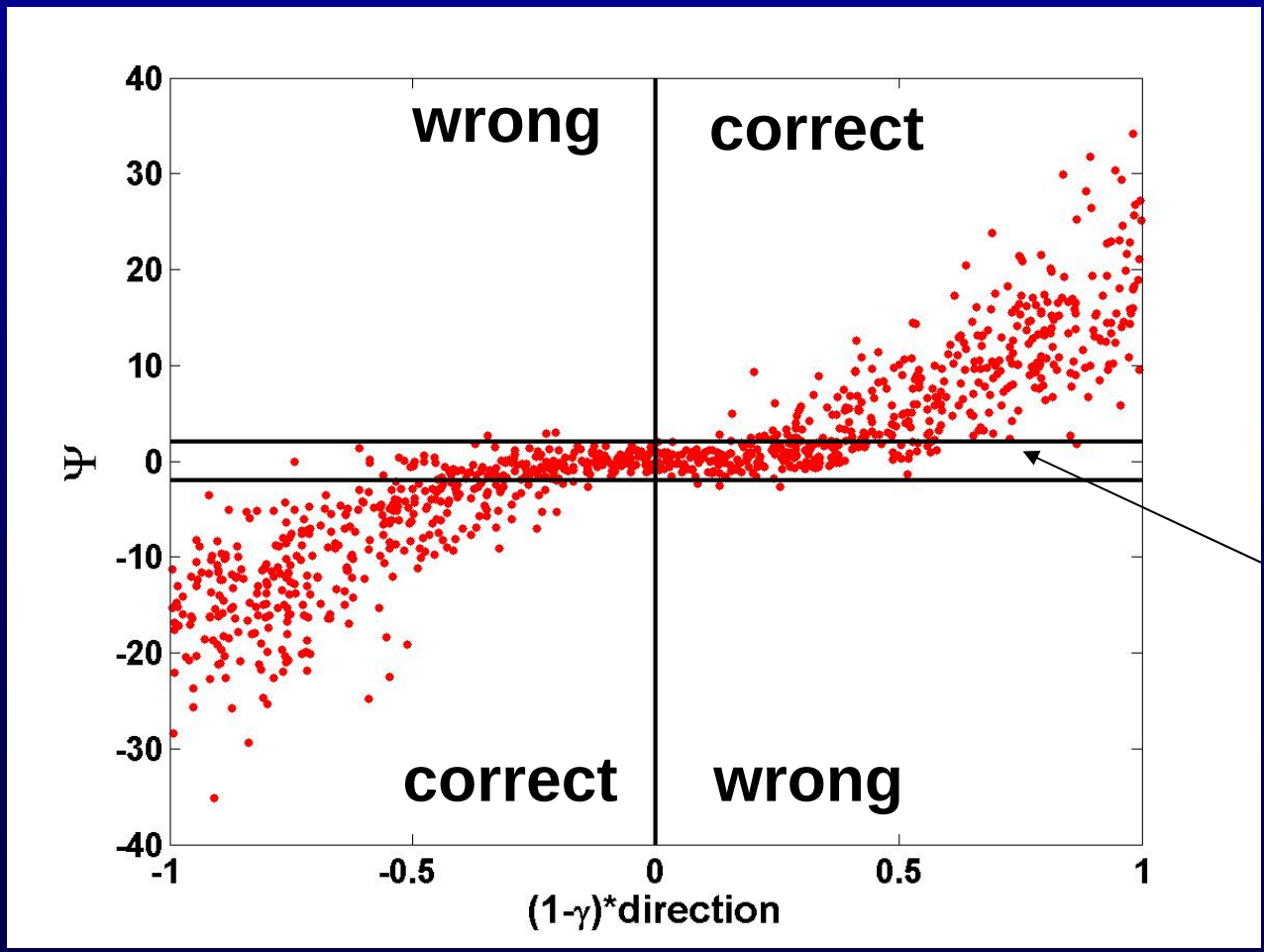
Correct	wrong	Total points
736	100	-264



I don’t know

Results for PSI

Correct	wrong	Total points
638	6	578



I don’t know

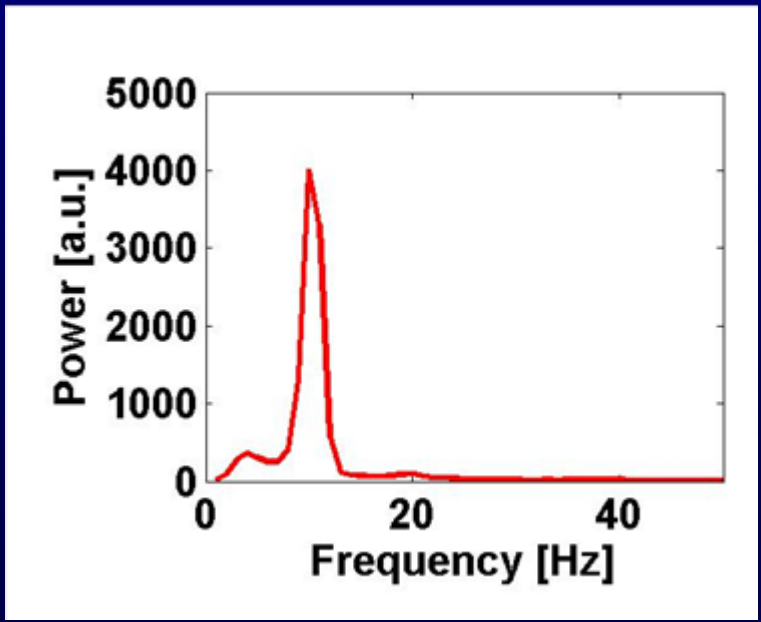
Real Challenge Data

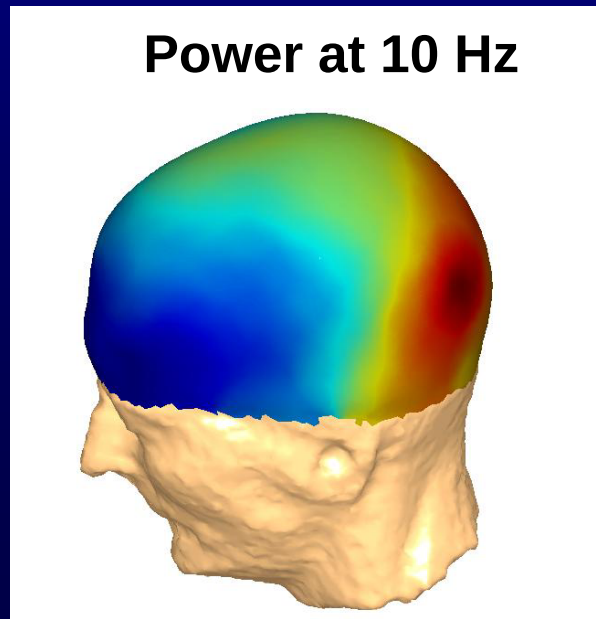
Description:

- 10 subjects
- eyes closed at rest
- ≈ 10 minutes each
- 256 Hz sampling rate
- 19 sensors

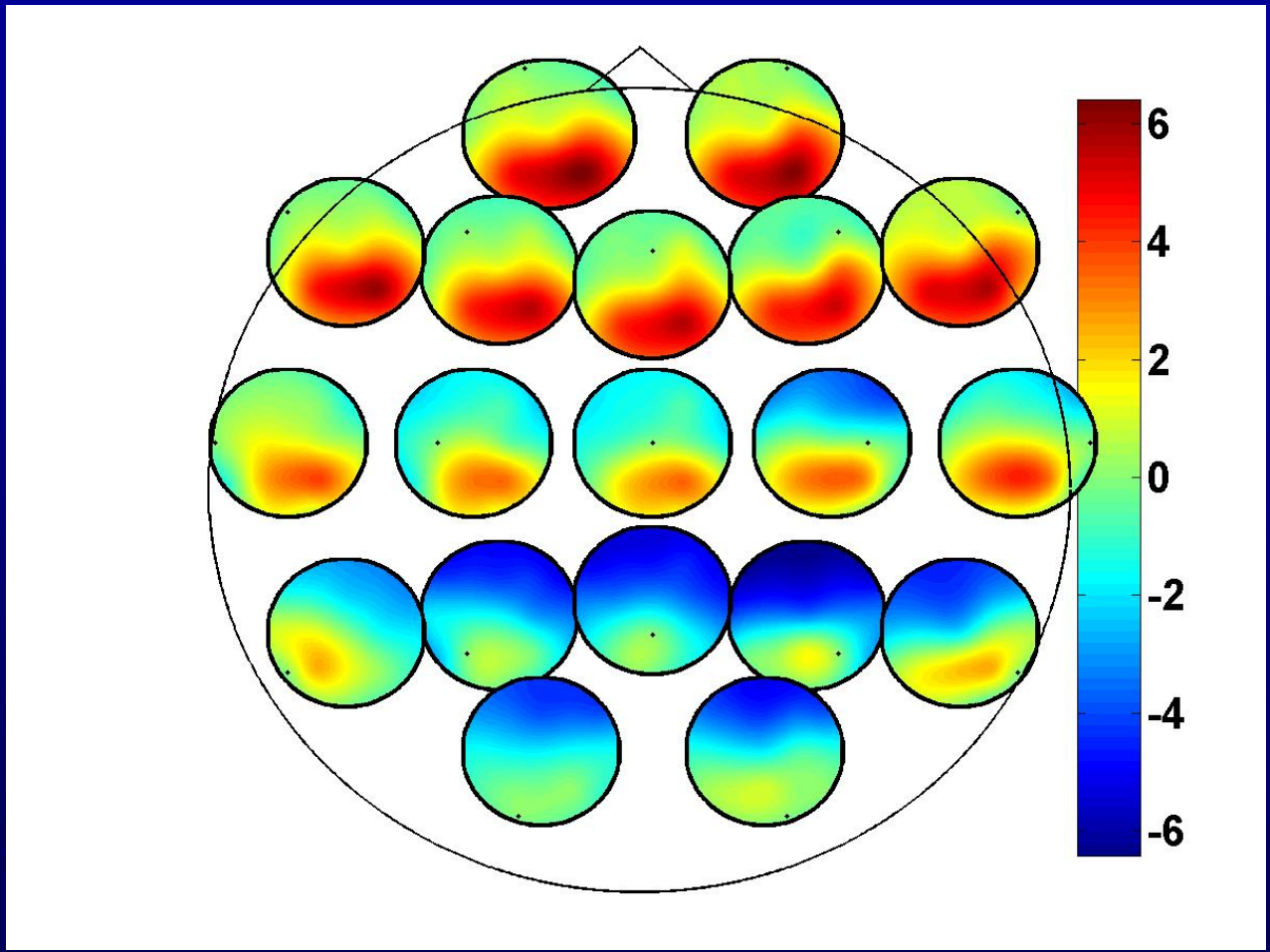
Features:

- strong „alpha rhythm“
- direction of alpha rhythm?





Results for PSI



Matlab code to create figures is provided at

http://ida.first.fraunhofer.de/~nolte/causality_challenge.html

Conclusion

What matters:

Simulated challenge data:

- **problem is generic (details are open to discussion)**
- **evidence is weighted**

Real challenge data:

- **excellent data (thanks to Tom Brismar)**
- **truth unknown**

Thanks to

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