Random Trees and Genealogies

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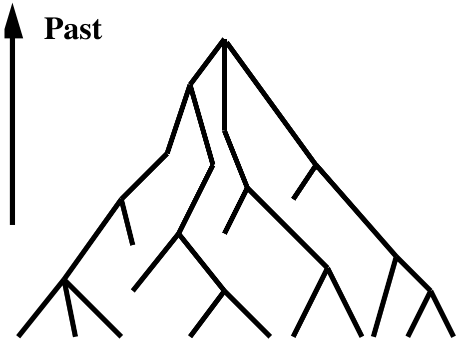
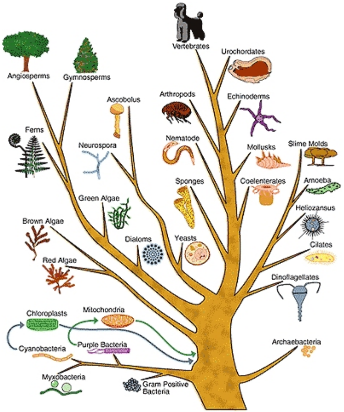
Jerusalem September 2008

OUTLINE

Asexual Reproduction



phylogeny



family names

bacteria

languages ...

Sexual Reproduction

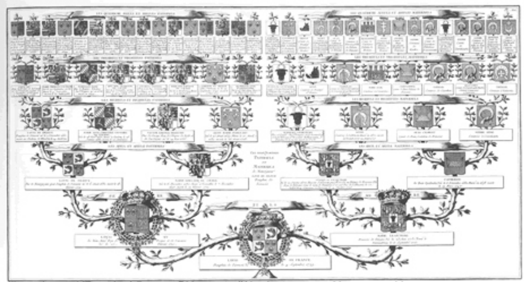
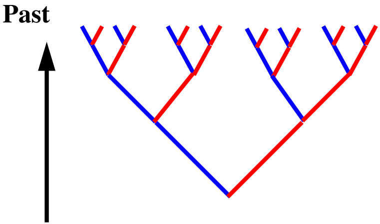
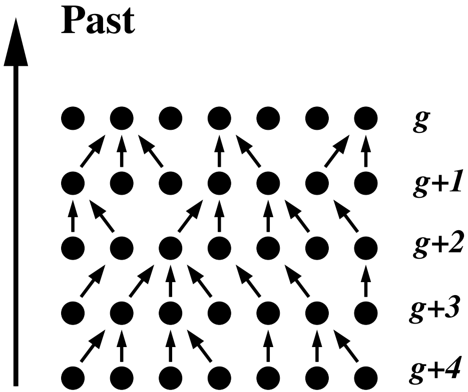


Illustration : *L'Encyclopédie,* par Diderot et d'Alembert - www.blason-armoiries.org



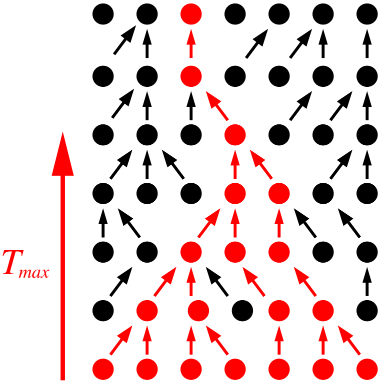
Wright-Fisher model (1930-1931)

- One parent model (asexual reproduction)
- Population of fixed size N
- Each individual has its parent chosen at random in the previous generation (neutrality)



Coalescence times:

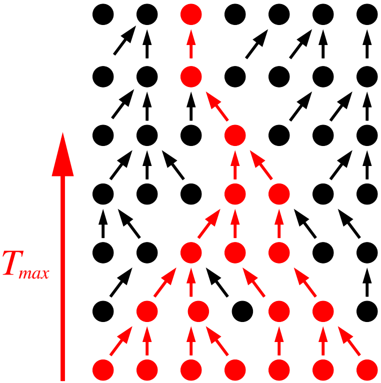
Age of the most recent common ancestor T_{max}



T_{max} depends on g

Coalescence times:

Age of the most recent common ancestor T_{max}



T_{max} depends on g

$$\langle T_{max} \rangle \simeq 2N$$

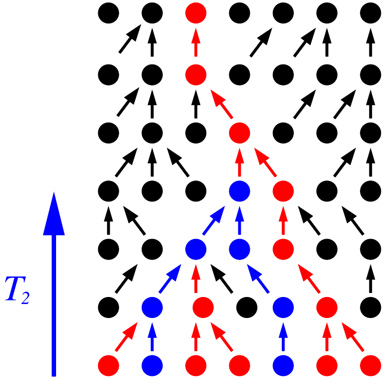
$$\langle {T_{max}}^2 \rangle - \langle T_{max} \rangle^2 \simeq C \; N^2$$

where $C = \frac{4\pi^2}{3} - 12 \simeq 1.6$

T_{max} is a non self-averaging quantity

Coalescence times:

Age T_2



The time T_2 depends on g and on the pair of individuals

Its average $\overline{T}_2$ over the population still depends on g

$$\langle T_2 \rangle \simeq N$$

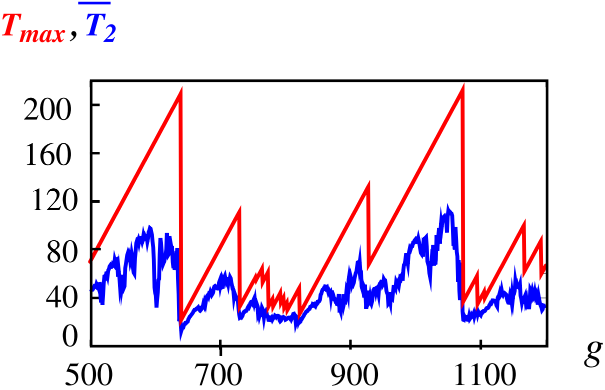
$$\langle {T_2}^2 \rangle - \langle T_2 \rangle^2 \simeq N^2$$

Two sorts of averages: $\begin{cases} \text{over the population} \\ \text{over } g \end{cases}$

Evolution of T_{max} and $\overline{T}_2$

T_{max} = age of the most recent common ancestor

$\overline{T}_2$ = average over the population of T_2

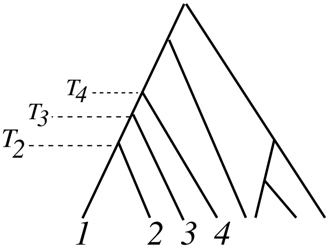


Serva 2005
Simon D. 2006

Coalescence times:

Age T_p

T_p = age of the most recent common ancestor
of p individuals chosen at random



$$\langle T_p \rangle \simeq \frac{2(p-1)}{p} N$$

(e^{-T_2} plays a role very similar to overlaps in spin-glasses)

Kingman's coalescent

Kingman 1982

$Q(k,k')$ = probability that k individuals have k' ancestors at the previous generation

$$Q(k,k) = \left(1-\frac{1}{N}\right)\left(1-\frac{2}{N}\right)\cdots\left(1-\frac{k-1}{N}\right)$$

$$Q(k,k') \simeq \begin{cases} 1-\frac{k(k-1)}{2N} & \text{for } k'=k \\ \frac{k(k-1)}{2N} & \text{for } k'=k-1 \\ O\left(\frac{1}{N^2}\right) & \text{for } k'<k-1 \end{cases}$$

Therefore

$$p \underset{t_p}{\longrightarrow} p-1 \underset{t_{p-1}}{\longrightarrow} \cdots 2 \underset{t_2}{\longrightarrow} 1$$

and

$$\boxed{T_p = t_p + t_{p-1} + \ldots t_2}$$

The times t_p are independent and have exponential distibutions with $\langle t_p \rangle = 2N/p(p-1)$

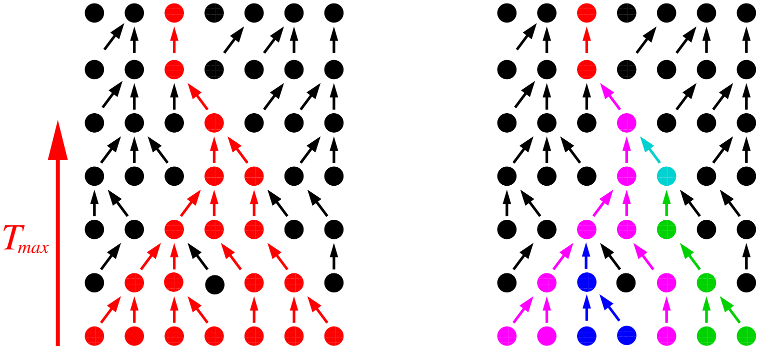
Beyond mean-field models

- One parent model
- Population of fixed size N
- Individuals are on the sites of a cube of $N = L^d$ lattice sites
- Each individual has its parent chosen at random in the previous generation in its neighborhood

Brunet D. Simon 2008

	$d \geq 2$	$d = 1$
$\frac{T_3}{T_2}$	$\frac{4}{3}$	$\frac{7}{5}$
$\frac{T_4}{T_2}$	$\frac{3}{2}$	$\frac{8}{5}$
$\frac{T_p}{T_2}$	$\frac{2(p-1)}{p}$	$\frac{2(p-1)(p+4)}{(p+1)(p+2)}$
$T_p \sim$	N	N^2

Genetic diversities



The sampling theory of selectively neutral alleles Ewens 1972

Summary on neutral models of asexual reproduction

- The coalescence times T_p are not selfaveraging
- Kingman's coalescent
- $T_p \sim N$
- Universal ratios $\frac{T_p}{T_2}$
- More is different

Summary on neutral models of asexual reproduction

- The coalescence times T_p are not selfaveraging
- Kingman's coalescent
- $T_p \sim N$
- Universal ratios $\frac{T_p}{T_2}$
- ~~More is different~~
- $d = 1$ is different (non-mean field)

Tomorrow's lecture: Evolution with **selection**

	neutral	selection
T_3/T_2	$4/3$	$5/4$
T_4/T_2	$3/2$	$25/18$
T_p/T_2	$2(p-1)/p$	long expression
Trees	Kingman 82	Parisi (79-80) Bolthausen-Sznitman (98)
$T_p \sim$	N	$(\log N)^3$

Sexual Reproduction

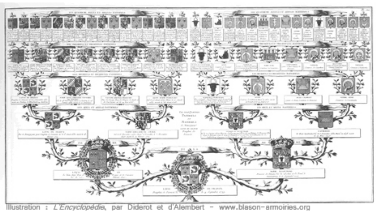
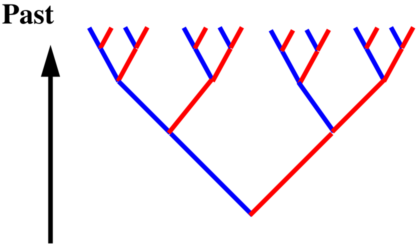


Illustration : *L'Encyclopédie*, par Diderot et d'Alembert – www.blason-armoiries.org

2 parents
4 grand parents
...
2^g ancestors g generations ago

At Roman times: $g \sim 76$

$$2^g \sim 10^{23}$$

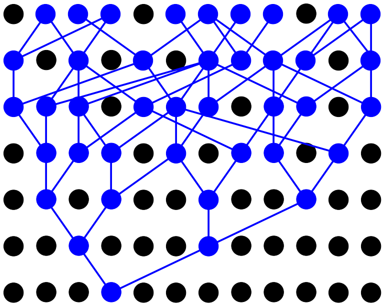


$$10^{23} \sim \frac{\text{mass of earth}}{\text{mass of man (of my size)}}$$

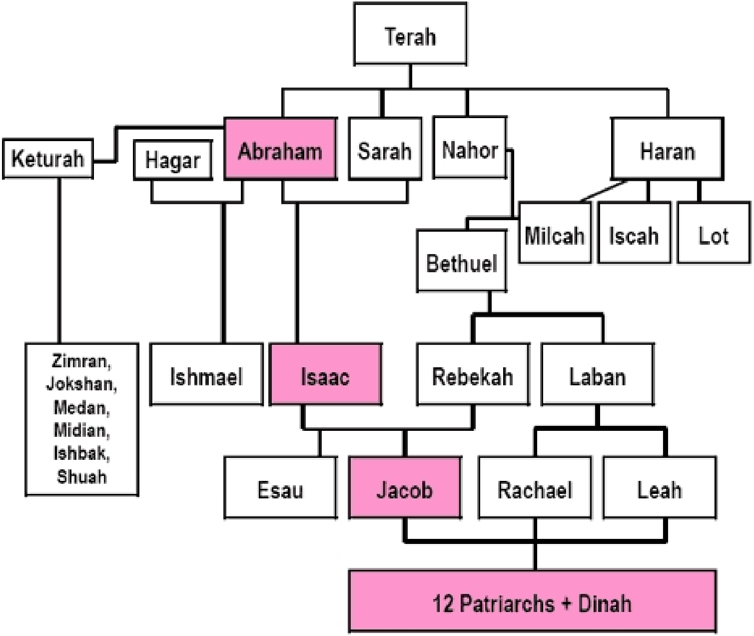
$\sim$ Avogadro's number

Neutral model

- Two parent model (sexual reproduction)
- Population of fixed size N
- Each individual has its two parents chosen at random in the previous generation (neutrality)



Abraham's Family Tree



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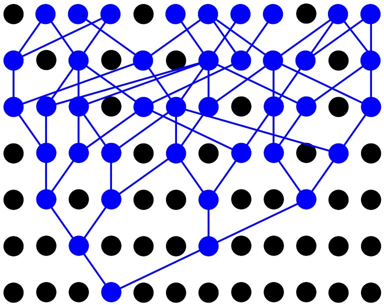

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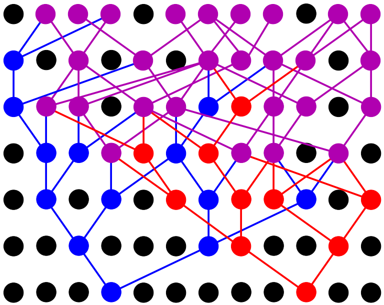


In the long term:

$$\begin{cases} \text{About} \sim .203N \text{ have no long term descendance} \\ \text{About} \sim .797N \text{ become ancestors of every body} \end{cases}$$

Chang 1999

Ages of ancestors: $N \sim 5 \times 10^9$

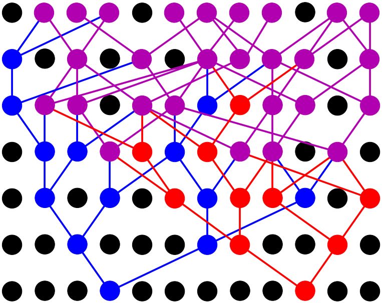
- Number of generations to at least one common ancestor of the whole population

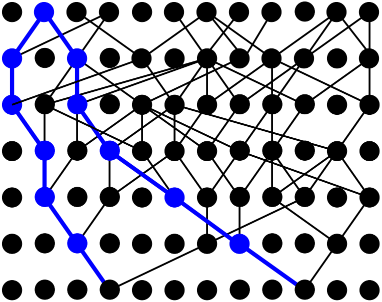
$$1.44 \log N \simeq 32$$

- Number of generations for all individuals to have all their ancestors in common

$$2.55 \log N \simeq 57$$

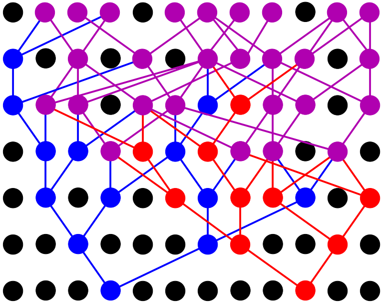
Neutral model

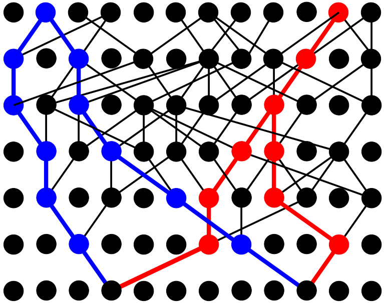




Abraham

Neutral model





Abraham

Lucy

An artist's view