Generalized Models of Dynastic Cycles

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The Dynastic Cycle

![Graph showing population changes over centuries.](image)

Peter Turchin: *Historical Dynamics*, 2003
Dynastic Cycle

Rulers → Law → Bandits → Mortality

Taxes

Crime

Production

Farmers → Mortality

Chu and Lee, *J. Pop. Econ* (1994)
The Challenge:

Study large, heterogeneous networks ...

... based on limited information ...

... to extract qualitative information ....

... on the long-term dynamics.
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Study large, heterogeneous networks... 
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Generalized Models

Dynastic Cycle

Correlation Analysis

Bifurcation Diagram

Four-Trophic Food Chain

Equivalence of Food Webs

Mitochondrial TCA-cycle

Niche Model
Cartoon

Gross et al. PRE 73, 016205, 2006.
Differential Equation

\[ \dot{X} = P(X) - L(X) \]
Generalized Models

Illustrative Example

Cartoon

\[ \dot{X} = P(X) - L(X) \]

Differential Equation

Assume: Steady State

\[ X^*, P^* = P(X^*), L^* = L(X^*) \]
Generalized Models

Illustrative Example

Cartoon

Differential Equation

\[ \dot{X} = P(X) - L(X) \]

Assume: Steady State

\[ X^*, P^* = P(X^*), L^* = L(X^*) \]

Define: Normalized Variables

\[ x = \frac{X}{X^*}, p(x) = \frac{P(X)}{P^*}, l(x) = \frac{L(X)}{L^*} \]
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Define: Normalized Variables

\[ x = \frac{X}{X^*}, \quad p(x) = \frac{P(X)}{P^*}, \quad l(x) = \frac{L(X)}{L^*} \]

Normalized System

\[ \dot{x} = \frac{P^*}{X^*} p(x) - \frac{L^*}{X^*} l(x) \]
Generalized Models

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Differential Equation

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Assume: Steady State

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Define: Normalized Variables

\[ x = \frac{X}{X^*}, p(x) = \frac{P(X)}{P^*}, l(x) = \frac{L(X)}{L^*} \]

Identify: Parameters

\[ \alpha = \frac{P^*}{X^*} = \frac{L^*}{X^*} \]
Generalized Models

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Identify: Parameters
\[ \alpha = \frac{P^*}{X^*} = \frac{L^*}{X^*} \]

Compute: Jacobian
\[ J = \alpha(p'(1) - l'(1)) \]
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Identify: Parameters
\[ \alpha = \frac{P^*}{X^*} = \frac{L^*}{X^*} \]
\[ \phi = p'(1) \]
\[ \lambda = l'(1) \]

Compute: Jacobian
\[ J = \alpha (\phi - \lambda) \]
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Differential Equation

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Assume: Steady State

\[ X^*, P^* = P(X^*), L^* = L(X^*) \]

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\[ x = \frac{X}{X^*}, \quad p(x) = \frac{P(X)}{P^*}, \quad l(x) = \frac{L(X)}{L^*} \]

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\[ \alpha = \frac{P^*}{X^*} = \frac{L^*}{X^*} \]

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Compute: Jacobian

\[ J = \alpha(\phi - \lambda) \]
Dynastic Cycle

Chu and Lee, *J. Pop. Econ* (1994)
A generalized model of the dynastic cycle

\[ \begin{align*}
\dot{F} &= P(F) - C(F, B) - T(F, R) - M(F) \\
\dot{B} &= C(F, B) - L(R, B) - M(B) \\
\dot{R} &= C(F, B) - M(R)
\end{align*} \]

Gross and Feudel, Phys Rev E 73 016205-14, 2005
The Dynastic Cycle

\[
\begin{align*}
\dot{F} &= P(F) - C(F, B) - T(F, R) - M(F) \\
\dot{B} &= C(F, B) - L(R, B) - M(B) \\
\dot{R} &= C(F, B) - M(R)
\end{align*}
\]

Assume that there is a **steady state**, then we can define

\[
f := \frac{F}{F^*}, \quad p(f) := \frac{P(F)}{P(F^*)}, \quad ...
\]

Gross and Feudel, Phys Rev E 73 016205-14, 2005
Generalized Models

The Dynastic Cycle

\[ \dot{f} = \frac{P^*}{F^*} p(f) - \frac{C^*}{F^*} c(f, b) - \frac{T^*}{F^*} t(f, r) - \frac{M^*}{F^*} m(f) \]

\[ \dot{b} = \frac{C^*}{B^*} c(f, b) - \frac{L^*}{B^*} l(r, b) - \frac{M^*}{B^*} m(b) \]

\[ \dot{r} = \frac{C^*}{R^*} c(f, b) - \frac{E^*}{R^*} e(r) \]

Gross and Feudel, Phys Rev E 73 016205-14, 2005
The Dynastic Cycle

\[ \begin{align*}
    \dot{f} &= \alpha_f (\rho - (\beta c - (1 - \beta)t)\rho - (1-\rho)m) \\
    \dot{b} &= \alpha_b (c - \gamma l - (1 - \gamma)m) \\
    \dot{r} &= \alpha_r (c - m)
\end{align*} \]

We have defined the scale parameters

\[ \alpha_f = \frac{P^*}{F^*} \] inverse life expectancy of farmers

\[ \gamma = \frac{1}{\alpha_b \frac{L^*}{B^*}} \] fraction of bandits that get eventually caught

Gross and Feudel, Phys Rev E 73 016205-14, 2005
Correlation Analysis

Parameter correlations with stability:

- $\alpha_y$, $\alpha_z$, $\beta_x$, $\beta_y$, $\gamma$, $c_x$, $c_y$, $l_y$, $l_z$, $m_y$, $n_x$, $r_z$, $s_x$, $t_x$, $t_z$
Bifurcation Diagram

Gross and Feudel, Phys Rev E 73 016205-14, 2005
Bifurcation Diagram

Gross and Feudel, Phys Rev E 73 016205-14, 2005
Bifurcation Diagram

Complex Dynamics

Gross and Feudel, Phys Rev E 73 016205-14, 2005
Generalized Models
Chaos in Food Chains

Four-Trophic Food Chain

Species 4
Species 3
Species 2
Species 1

Gross, Ebenhöh and Feudel, Oikos 109(1) 135-155, 2005
Chaos in Food Chains

Chaotic Dynamics

Gross, Ebenhöh and Feudel, Oikos 109(1)135-155, 2005

Thilo Gross - Dynamics of Biological Networks - Max-Planck Institut für Physik komplexer Systeme, Dresden
Equivalence of Food Webs
Generalized Models

A Model of Glycolysis

\[
\begin{align*}
\text{ATP} & \xrightarrow{v_8} \text{ADP} \\
\text{Glc} & \xrightarrow{v_1} \text{FBP} \\
2 \text{ATP} & \xrightarrow{v_7} 2 \text{ADP} \\
\text{TP} & \xrightarrow{v_3} \text{NADH} \\
\text{BPG} & \xrightarrow{v_4} \text{NADH} \\
\text{Pyr} & \xrightarrow{v_5} \text{EtOH}
\end{align*}
\]

Steuer, Gross, Selbig & Blasius
PNAS 103 (2006)
Mitochondrial TCA-cycle

Mitochondrial TCA-cycle


No stable stationary states for mass action kinetics
Generalized Models

Mitochondrial TCA-cycle


Pyruvate import is destabilizing

No stable stationary states for mass action kinetics
Mitochondrial TCA-cycle


Weak Saturation of NAD-malic reaction

Strong saturation of malatedehydogenase
Generalized Models

Mitochondrial TCA-cycle


Weak Saturation of NAD-malic reaction

Strong saturation of malatedehydrogenase
Generalized Models

Niche Model Food Webs

Niche Model

Species

Connectance

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Generalized Models

Niche Model

Species

35 Billion food webs

Connectance

Niche Model Food Webs

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Conclusion

Generalized modeling should be used as a **high-throughput screening tool** before conventional modeling is attempted.

It is particularly useful if **qualitative information** on the **local dynamics** is desired.
Thank you very much for your attention