Democratic Approximation of Lexicographic Preference Models

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Preference Learning

- Suppose we know some features of flights and some examples of how a customer chose their flights.
  - Good Food
  - Good Price
  - Few Cancelations
  - Good Service
  - Data Access

0	Chosen over	1
1		1
1		0
0		1
0		1

- Can we determine which of these flights the same customer will choose?

0	1	1	0	1

Versus

1	0	0	1	0

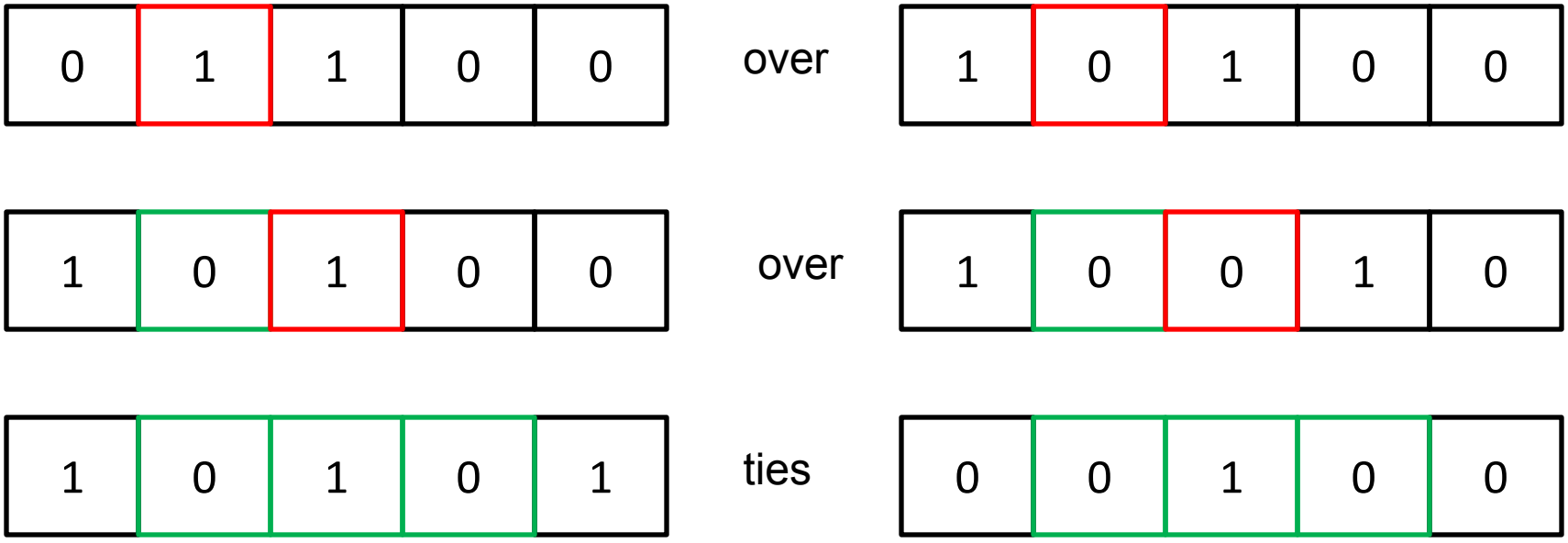
Lexicographic Preference Models

- LPMs are a basic preference model
  - *Few Cancelations* over *Lots of Cancelations* (always)
  - *Good Price* over *Bad Price* (within "Few Cancelations" group)
- Formally, given objects described by n variables $[x_1 \ldots x_n]$, an LPM L on X is a *total order* $\sqsubseteq_L$ on a *subset* $R \subseteq X$
  - R = *relevant variables*, I = X – R = *irrelevant variables*
- Used in many research areas, including Multicriteria Optimization (Bertsekas & Tsitsiklis, 97), Linear Programming, and Game Theory (Quesada, 03)

LPM Decision Example

- Given 2 Objects, A and B and an LPM L
  - Find the smallest variable, X* in $\sqsubseteq_L$ Such that X* has different values in A and B and choose the one where X*=1, otherwise there is a tie.
- From our flight example, suppose we have

LPM L = X2 < X3 < X4 (price < cancels < service)

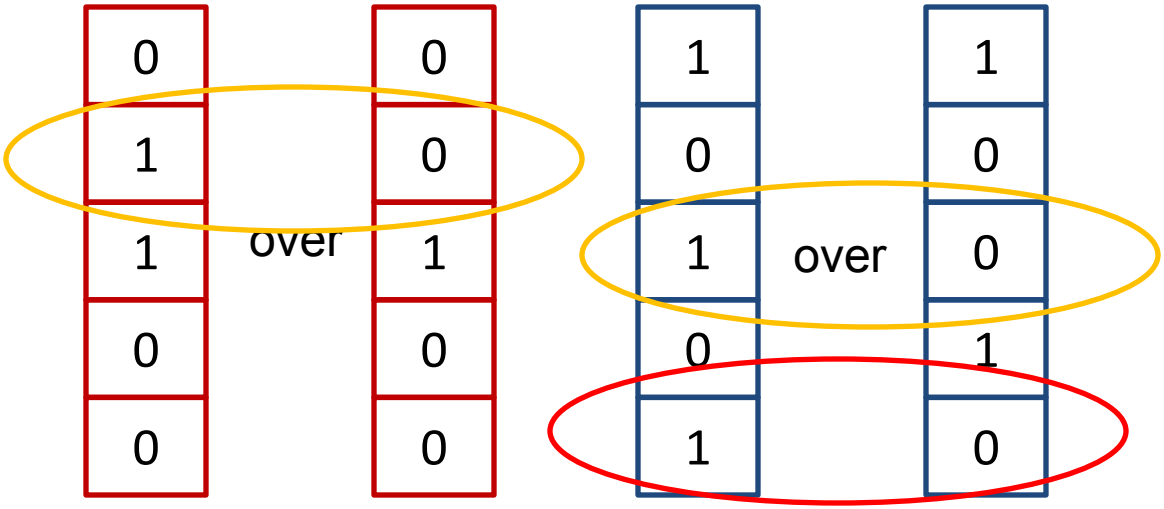


LPM Learning

- Given
  - Set of observations O, where o_i = (A, B), for A= [x_1…x_n] (also B=[x_1...x_n]), where A is preferred over B.
- Find
  - An LPM L that is consistent with O.
    - L implies A is preferred over B or that they are equally preferred.
- Finding a consistent LPM was studied in previous work…

State of The Art

- The Greedy Approach (Schmitt and Martignon, 2006)
  - Iteratively construct L, choose a variable that creates the fewest conflicts with the observed data (break ties randomly).
- Drawback: This algorithm only gives us 1 of *many possible* consistent LPMs.
  - The bias in the random selection will greatly impact predictions performance.



{X2, X3}

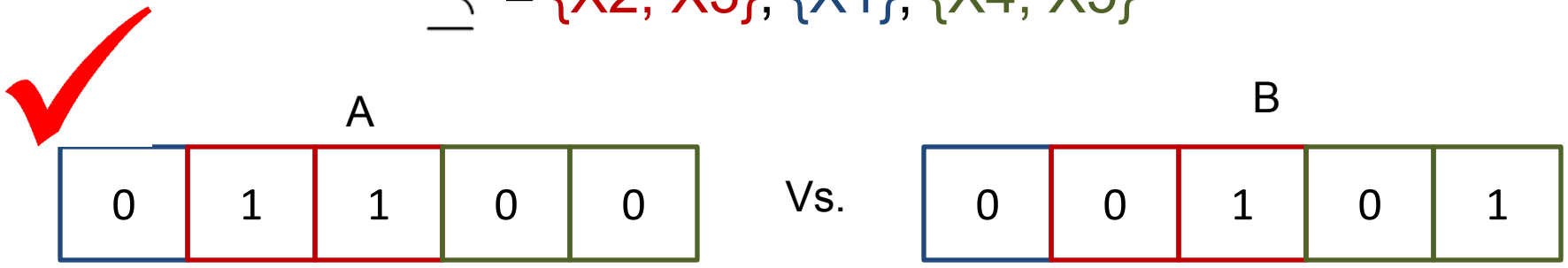
{X2, X5, X3}?

{X1, X5, X2}?

New Approach 1: Variable Voting

- Given a partial order $\preceq$ on X, vote among the choices for "deciding" variable.
  - D = variables that differ between A and B
  - D* = variables in D with smallest rank according to $\preceq$
  - Each variable in D* has one vote.

$\preceq$ = {X2, X3}, {X1}, {X4, X5}



D = X2, X5

D* = X2

X2=1 for A, so A is preferred over B.

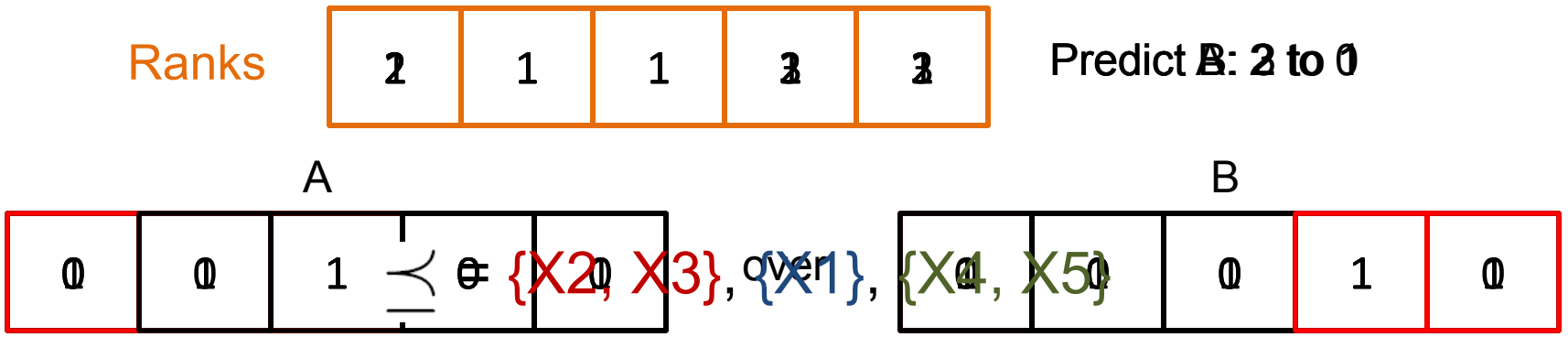
Learning Variable Ranks

- Algorithm *LearnVariableRank* (*LVR*)

  Initially all variables have rank 1 *(most important)*

  While the ranks can change based on the observations

  - **For each** observation, predict a preferred object.
    - **If** the prediction is wrong, increment the ranks of all the variables that predicted incorrectly.



Variable Vote Properties

- Correctness
  - All $\sqsubseteq$ that are a topological sort of $\succ$ from *LVR* are consistent with O. *LVR* never increments the ranks of relevant variables beyond their actual rank.
- Convergence
  - Mistake bound of $O(n^2)$. If all variables are relevant, *LVR* converges to the true LPM. With irrelevant variables, it converges to a correct ordering of the relevant variables.
- Time Complexity
  - $O(n^3m)$ (compared to $O(n^2m)$ for Greedy)

New Approach 2: Model Voting

- A Bayesian Approach – look at a sample of consistent LPMs and let each one vote (equally weighted)
- For a set of LPMs S (consistent with O), choose A over B if

$$\sum P(L|S) * V_{A>B}^{L} \; > \; \sum P(L|S) * V_{B>A}^{L}$$

- Aggregate LPMs (X3, X2, *) allow us to compactly represent sets of models.

Generating the Models

- Algorith *SampleModels*

  Candidates = {variables Y | Y $\notin$ rulePrefix, $\forall$ (A, B) $\in$ O, A(Y) = 1 or A(Y) = B(Y)

  While *Candidates* $\neq \emptyset$

    If *O* = $\emptyset$ then **return** (***rulePrefix***, *)

    Randomly choose variable ***Z*** from ***Candidates***

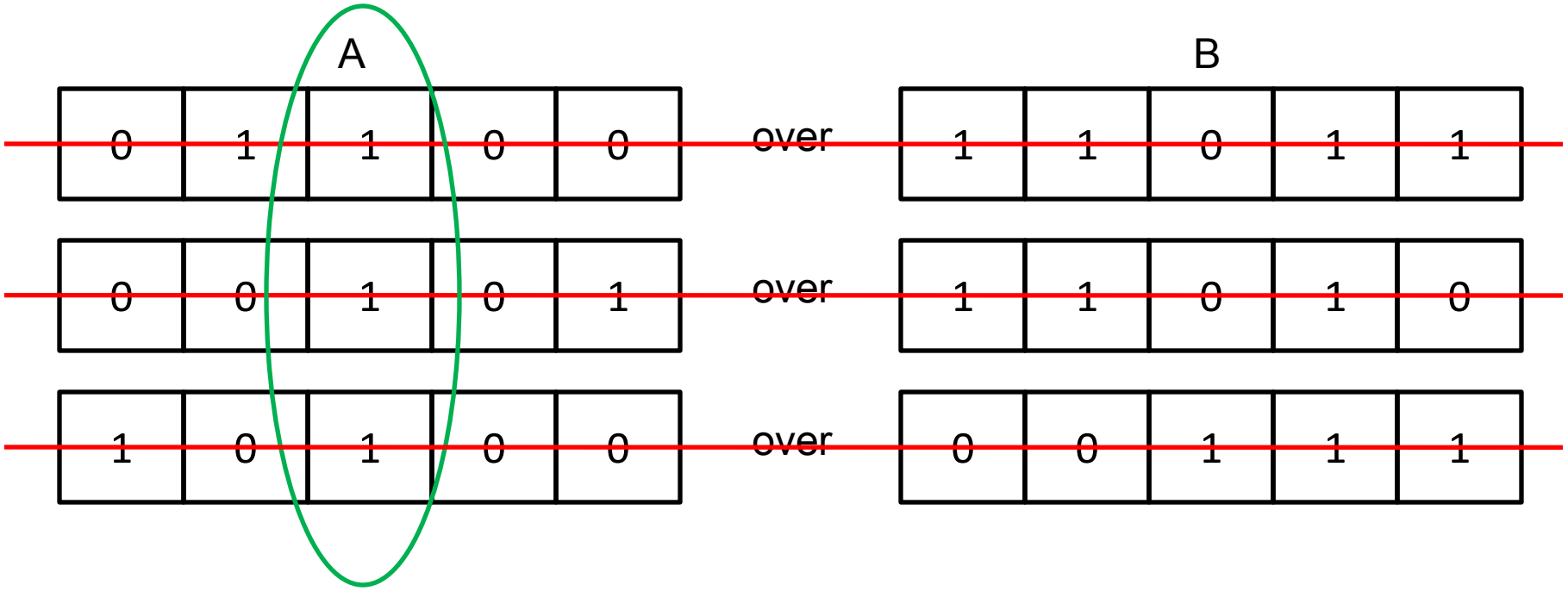
    Remove any ***(C, D)*** $\in$ ***O*** where ***C(Z)*** $\neq$ ***D(Z)***

    rulePrefix = (***rulePrefix***, ***Z***)

    Recompute ***Candidates***

  return *rulePrefix*

Example Model Generation



Candidates = {X3, X2}

model = (X3, X2, X1, *)

Model Vote

Given a set of samples S and objects A and B

SampSize = total number of models represented in S
(computation is linear in n and |S|)

For each L ∈ S

If L is non-aggregate

Vote(winner) += 1

Else If L is aggregate and one of the prefix variables determines a winner

Vote(winner) += number of extensions

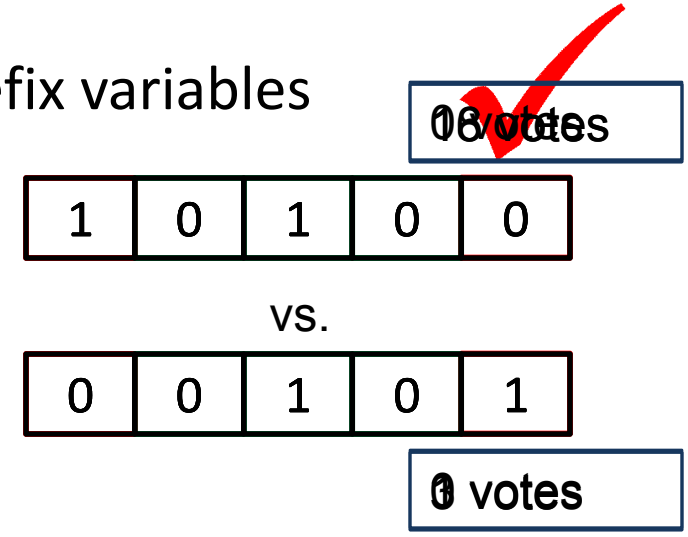
Else (L is aggregate, split decision)

Vote(A) += $N_{A>B}^{L}$ (linear time)

Vote(B) += $N_{B>A}^{L}$ (linear time)

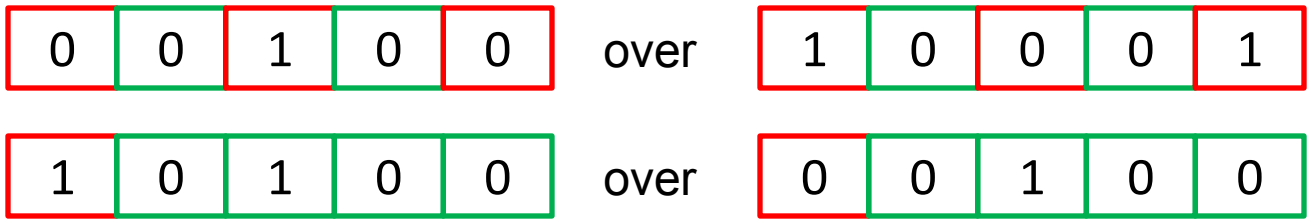
Return the object with the most votes

model = (X3, X2, X5, X1, X4)



Model Vote Properties

- Complexity
  - *SampleModels*: $O(n^2m)$
  - With s calls to *SampleModels*, *ModelVote* takes O(sn) time.
    - All the numerical computations are linear with proper caching.
- Comparison to *Variable Voting*
  - *Model Vote* considers LPMs *Variable Vote* cannot, such as (X3, X1, *) to explain the following.



    - VV never considers X1 < X2 because X2 is never wrong.
    - It seems like this would be helpful with irrelevant variables…

Introducing Bias

- We can limit the number of models considered by using a *bias* based on *domain knowledge.*
- When buying a used car, people consider
  - {*mileage, year, make*} before
  - {*color, #doors, body type*} before
  - {*interior color, cup holders*}
- A learning bias B is a total order on a *partition* of the variables.

  {X1, X2, X3} < {X4, X5} < {X6, X7, X8, X9, X10}

- 9 variables → 905,970 LPMs
  - 3 partitions of 3 variables → 646 LPMs

Voting Algorithms and Bias

- Variable Vote can easily be extended to use bias by initializing the variables based on the bias.

{X1, X2, X3} < {X4, X5}

1	1	1	4	4

- Model Vote can use a bias by
  - Change *SampleModels* to only produce bias-consistent models.
  - For aggregate LPMs, compute the number of *bias-consistent* extensions that vote for each object.
    - This can still be computed efficiently (details in the paper).

Experimental Setup

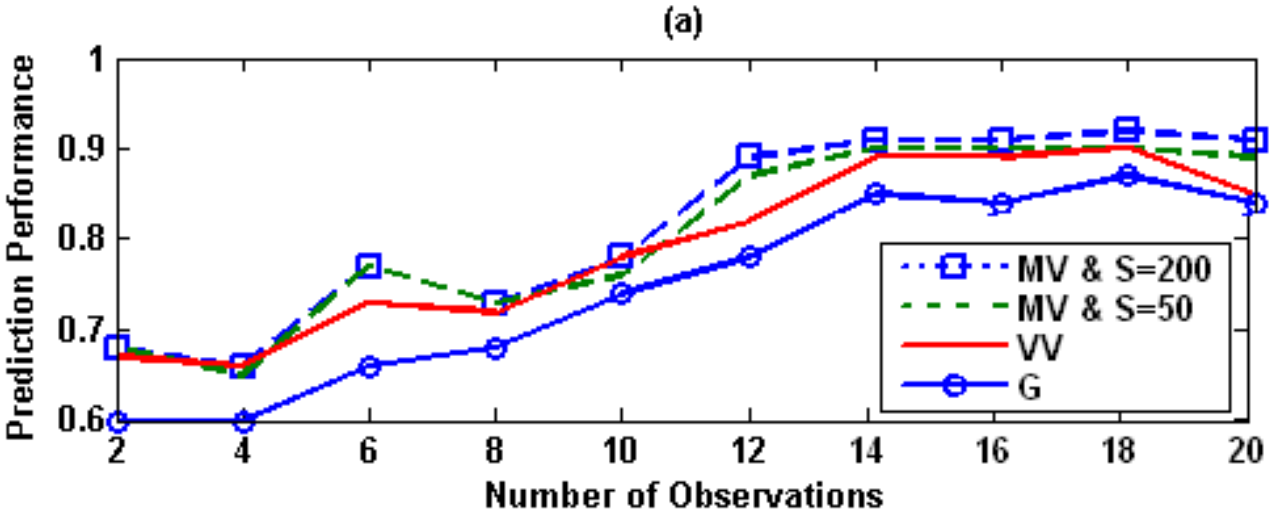
- Prediction Performance on Test Set *T*

Performance(T) = (Correct(T) + .5 * Tie(T)) / |T|

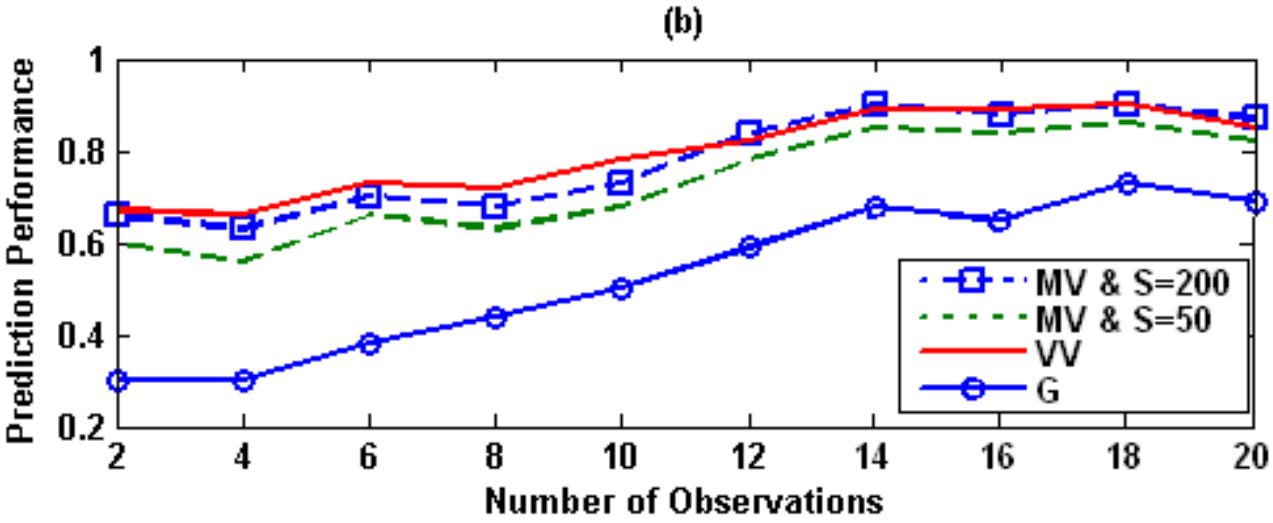
- Concerned with Average and worst results for each algorithm on the test sets.
- Mitigating randomness
  - Ran *Greedy* 200 times for every (O, T) pair
  - Ran *MV* 10 times for each (O, T) pair, varied number of calls to sample models (S=50, S=200)

Empirical Comparison

Average Case



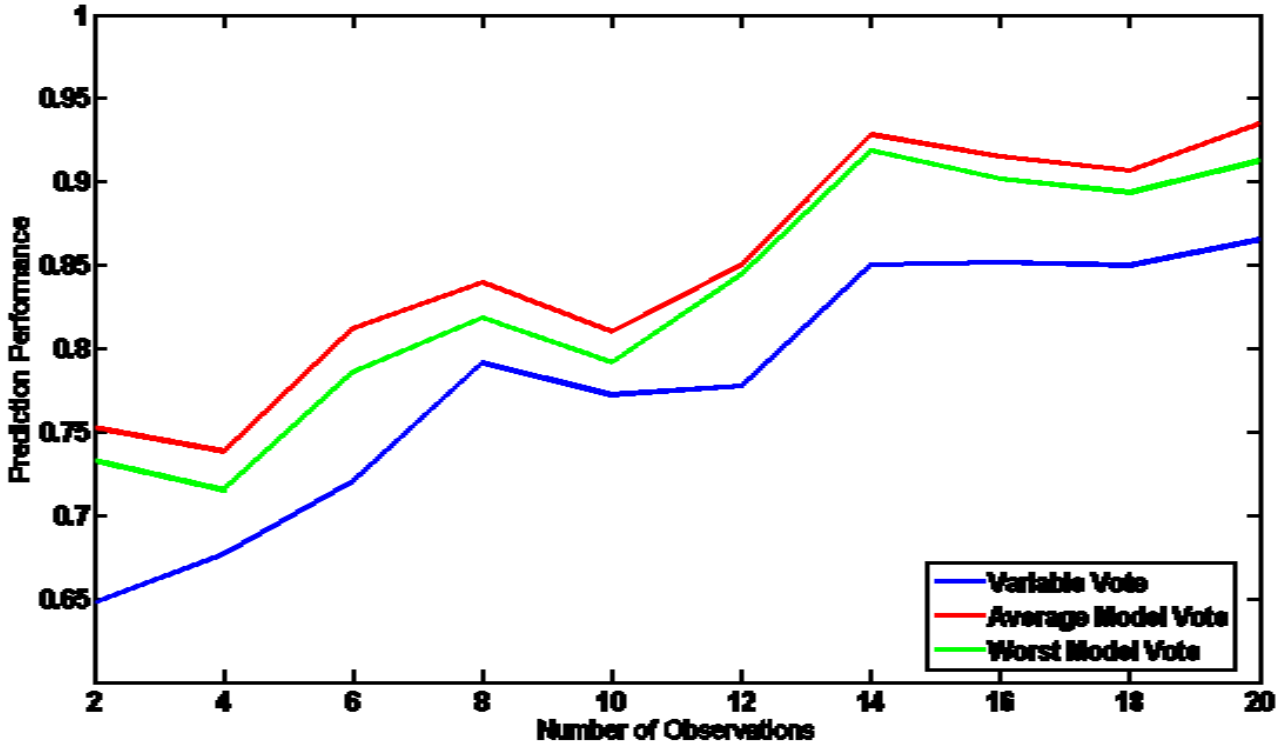
Worst Case



|R| = 15, |I| = 0, |T| = 20, data points average over 20 sets

Model Voting and Irrelevant Variables

- As expected, MV outperforms VV in the presence of irrelevant variables



|R| = 5, |I| = 10, |T| = 20, data points average over 20 sets

May want to mention G worst case

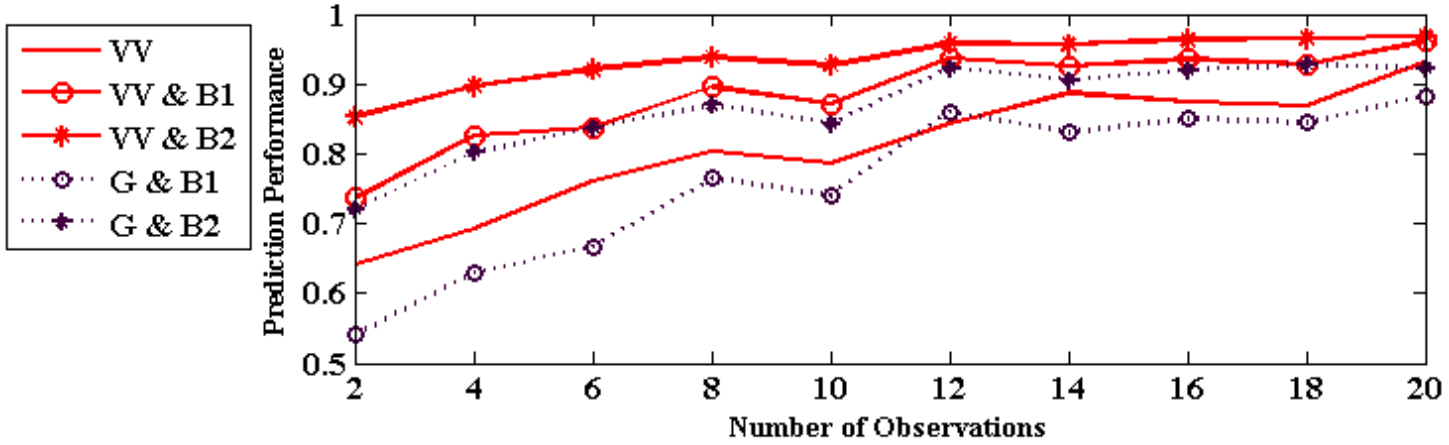
Bias Experiments

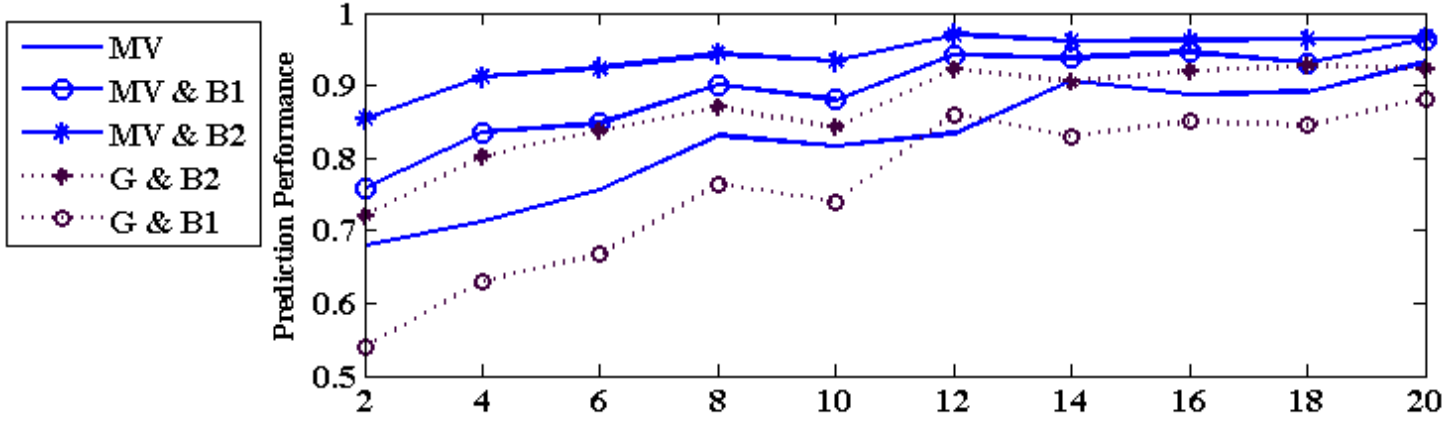
B1 = {X1, X2, X3, X4, X5} < {X6, X7, X8, X9, X10}

B2 = {X1, X2, X3} < {X4, X5} < {X6, X7, X8, X9, X10}

|R| = 15, |I|

|T| = 20





Conclusions & Future Work

- We developed algorithms that learn preference models, but do not commit to a single model.
  - Instead they predict based on a majority of the voting models
  - Our methods are theoretically sound and computationally efficient.
  - Out voting methods outperform the state of the art Greedy method in empirical studies with modest increases in computation.
- Future directions
  - Learning preferred values of variables.
  - Democratic approximation techniques for other types of preference models.