A few slides on Interior Point SVM entries

For PASCAL Largescale Learning Challenge workshop

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Linear SVM using interior point method

Primal formulation

$$\min_{\underline{w}, w_0, \underline{\xi}} \quad \frac{1}{2}\underline{w}^T\underline{w} + C\underline{e}^T\underline{\xi}$$

$$\text{s.t.} \quad Y(X^T\underline{w} + w_0\underline{e}) \geq \underline{e} - \underline{\xi}$$

$$\underline{\xi} \geq \underline{0}$$

Very many constraints $\Rightarrow \mathcal{O}(n^2)$

Dual formulation

$$\min_{\underline{\alpha}} \quad \frac{1}{2}\underline{\alpha}^T \, YX^TXY \, \underline{\alpha} - \underline{e}^T\underline{\alpha}$$

$$\text{s.t.} \quad \underline{y}^T\underline{\alpha} = 0$$

$$\underline{0} \leq \underline{\alpha} \leq C\underline{e}$$

Dense Hessian $\Rightarrow \mathcal{O}(n^3)$

Linear SVM using interior point method

Primal formulation

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Dual formulation

$$\min_{\underline{\alpha}} \quad \frac{1}{2}\underline{\alpha}^T \, YX^TXY \, \underline{\alpha} - \underline{e}^T\underline{\alpha}$$
$$\text{s.t.} \quad \underline{y}^T\underline{\alpha} = 0$$
$$\underline{0} \leq \underline{\alpha} \leq C\underline{e}$$

Dense Hessian $\Rightarrow \mathcal{O}(n^3)$

Separable formulation

$$\min_{\underline{w}, \underline{\alpha}} \quad \frac{1}{2}\underline{w}^T\underline{w} - \underline{e}^T\underline{\alpha}$$
$$\text{s.t.} \quad \underline{w} - XY\underline{\alpha} = \underline{0}$$
$$\underline{y}^T\underline{\alpha} = 0$$
$$\underline{0} \leq \underline{\alpha} \leq C\underline{e}.$$

Based on dual formulation.

Few constraints $\Rightarrow \mathcal{O}(m^2)$

Diagonal Hessian $\Rightarrow \mathcal{O}(n)$

Direct access to $\underline{w}$ and w_0 using primal-dual IPM

Efficient computations on multi-core processors

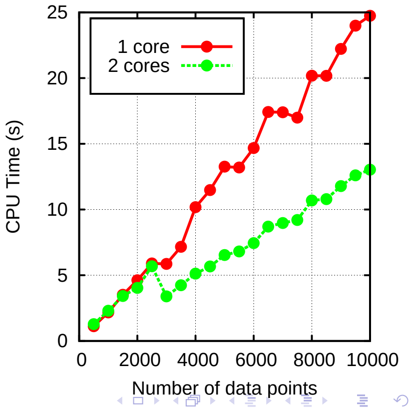
Vast majority of time ($>$ %95) is spent calculating matrix M:

$$M \equiv \begin{bmatrix} I_m & -XY \\ 0 & \underline{y}^T \end{bmatrix} \begin{bmatrix} I_m & 0 \\ 0 & \Theta_\alpha^{-1} \end{bmatrix}^{-1} \begin{bmatrix} I_m & 0 \\ -YX^T & \underline{y} \end{bmatrix}$$

Θ_α is diagonal matrix associated with logarithmic barrier.

Aim to minimize number of layer 2 cache misses:

- Divide data into “cache-sized” blocks
- Use BLAS Level 3 operations as much as possible (here DGEMM)
- BLAS libraries support shared memory threading across processor cores e.g. GotoBLAS, Intel MKL
- Layer 2 cache misses $\approx$ 2% of data reads



Termination based on hyperplane stability

- $\underline{w}$ (hyperplane) converges remarkably quickly
- $\underline{\alpha}$, duality gap and residual errors converge afterwards
- Use hyperplane stability for termination in version 2

Angle ϕ of hyperplane between iteration $i-1$ and i

$$\cos\phi = \frac{(\underline{w}^{(i-1)})^T \underline{w}^{(i)}}{\|\underline{w}^{(i-1)}\| \|\underline{w}^{(i)}\|}$$

Precision recall and change in hyperplane angle for alpha dataset: