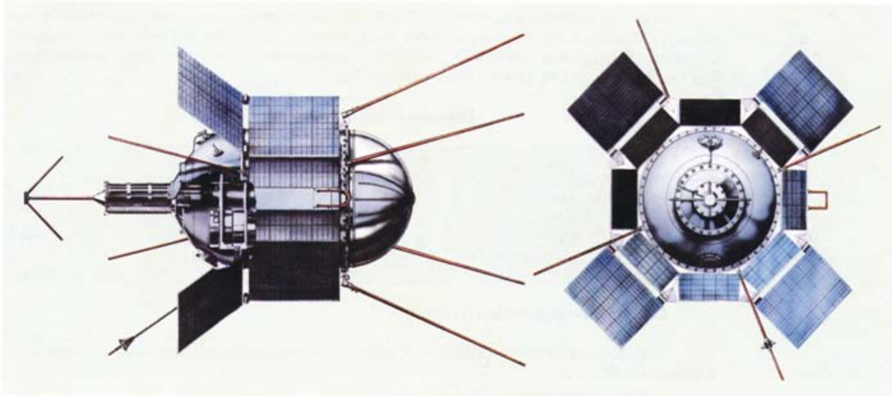
«Analysis of possibilities of Earth’s gravitational field measurements using global navigation satellite systems»

Matvienko S.A.

Yuzhnoye SDO

Ukraine

SPACECRAFT DS-U2-M (molecular)



General Specification

Mass, kg ………………………………………………..230
Calculated Orbit Parameters
Perigee Altitude, km………………………………..220
Apogee Altitude, km ……………………………...2100
Orbit Inclination, deg. ……………………………….49
Active Operation Period, days …………………………90
Launch Vehicle ……………………………………..11K63

Scientific equipment includes:
* Double-beam molecular generator MG-2 which operates in the mode of inversion area of ammonia with quantum number of j=3, κ=3 (generated frequency 23870,13 MHz, relative frequency instability not exceeds 1,0x10~11 per 60 min.);
* frequency matching unit E-149M;
* device E-150M;
* unit for temperature control system of resonator and ammonia tank MG;
* on-board receiver E-155M of ultra-high frequency bandwidth for signal reception and conversion of ground telemetry stations;
* unit for transmission device and switching of on-board equipment E-151 operating modes;

Launch date: November 26, 1965.

Spacecraft purpose: * test of operation and measurements of frequency stability of on-board molecular generator under space flight conditions;
* measurements of frequency gravitational shift of on-board molecular generator with the purpose of general relativity theory verification.

Radiophysical method

Total frequency shift of GNSS electromagnetic radiation

ΔfΣ = ΔfD + Δfg

where ΔfΣ – total frequency shift
ΔfD – Doppler frequency shift
Δfg – gravitational frequency shift

Gravitational shift of signal frequency

$$\frac{\Delta f_g}{f_0} = \frac{f_0 - f_1}{f_0} = \frac{u_0 - u_1}{c^2} \qquad \textbf{(1)}$$

where f – frequency
u – gravitational potential
c - speed of light

$$u = \frac{a_{00}}{R} = \frac{\gamma M}{R}$$

$\gamma M = 3{,}98 \cdot 10^5$ km^3/sec^2

$$\frac{\Delta f}{f_0} = \frac{\Delta u}{c^2} = \frac{\frac{a_{00}}{R_E + L} - \frac{a_{00}}{R_E}}{c^2} = \frac{\gamma M}{c^2}\left(\frac{1}{R_E + L} - \frac{1}{R_E}\right) = 4.46 \cdot 10^{-10} \qquad \textbf{(2)}$$

This value exceeds much on-board frequency standard of existing GNSS error value of 10-13. The above fact demonstrates principal technical feasibility of radiophysical method of acceleration of gravity value measurement implementation.

Well-known Pound-Rebka ratio is introduced below (awarded by the Nobel Prize in 1926)

$$\frac{\Delta f_g}{f_0} = \frac{g\Delta R}{c^2} \tag{3}$$

Comparing (2) and (3) we will obtain the following

$$gR = \frac{\gamma M}{R}$$

This is well-known equation for Newton's law of gravitation

$$F = mg = \frac{\gamma m M}{R^2}$$

However, for circular orbital motion under Earth's gravity one should use the following ratio

$$\frac{mV^2}{R_0} = mg \tag{4}$$

u = V2 = gR

If we substitute ratio (4) into ratio (3) we will obtain the ratio for ground-based measurements

$$\frac{\Delta f}{f_0} = \frac{u_0 - V_0^2}{c^2} \tag{5}$$

$$u_0 = V_0^2 + \frac{\Delta f}{f_0} c^2 \tag{6}$$

$$g = \frac{V_0^2 + \frac{\Delta f}{f_0} c^2}{H_0} \tag{7}$$

Such measurements method will be implemented under STCU Project #3856

Ratio for radiophysical method of measurement of gravitational field parameters is introduced in three variants:

1. **Measurement of frequency gravitational shift on SC board using GNSS signals**

$$\frac{\Delta f_g}{f_0} = \frac{V_0^2 - V_1^2}{c^2} \quad (8)$$

2. **Measurement of frequency gravitational shift on Earth using GNSS signals**

$$\frac{\Delta f_g}{f_0} = \frac{u_0 - V_1^2}{c^2} \quad (9)$$

3. **Measurement of frequency gravitational shift on Earth (Pound-Rebka experiment)**

$$\frac{\Delta f_g}{f_0} = \frac{u_0 - u_1}{c^2} = \frac{g \Delta H}{c^2} \quad (10)$$

One could calculate relative gravitational shift of solar system planets radiation relative to Earth using formula (8)

$$\frac{\Delta f_g}{f_0} = \frac{V_0^2 - V_1^2}{c^2}$$

Parameters / Planets	R 10^6 km	V_{orb} Km/sec	$\frac{\Delta f}{f_0}$ 10^{-10}
Mercury	58	48,8	-165,9
Venus	108	35,0	-37,4
Earth	149	29,8	
Mars	227,9	24,2	33,6
Jupiter	778,4	13,06	79,8
Saturn	1429,4	9,46	88,72
Uranus	287	6,81	93,51
Neptune	4498	5,432	95,39
Pluto	5906,4	4,666	96,25

Conclusion

- Technical characteristics of existing GNSS allow to determine gravitational potential and absolute value of acceleration of gravity
- It is necessary to use frequency standards with relative stability not less than 10-15 to determine gravitational field fluctuations at a level of 10 mGal
- Use of the proposed approach will enable additional GNSS functionality that will allow to solve not only geodesic tasks, but also a number of important applied issues, particularly:
  - gravitational minerals survey
  - seismic activity forecast
  - analysis of gravitational anomalies influence at psychophysical state of people