Analysis of possibilities of Earth’s gravitational field measurements using global navigation satellite systems

Matvienko S.A.
Yuzhnoye SDO
Ukraine
SPACECRAFT DS-U2-M (molecular)

Scientific equipment includes:
* Double-beam molecular generator MG-2 which operates in the mode of inversion area of ammonia with quantum number of j=3, κ=3 (generated frequency 23870.13 MHz, relative frequency instability not exceeds 1.0×10⁻¹¹ per 60 min.);
  * frequency matching unit E-149M;
  * device E-150M;
  * unit for temperature control system of resonator and ammonia tank MG;
  * on-board receiver E-155M of ultra-high frequency bandwidth for signal reception and conversion of ground telemetry stations;
  * unit for transmission device and switching of on-board equipment E-151 operating modes;
Launch date: November 26, 1965.

Spacecraft purpose: * test of operation and measurements of frequency stability of on-board molecular generator under space flight conditions;
* measurements of frequency gravitational shift of on-board molecular generator with the purpose of general relativity theory verification.

<table>
<thead>
<tr>
<th>General Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, kg</td>
</tr>
<tr>
<td>230</td>
</tr>
<tr>
<td>Calculated Orbit Parameters</td>
</tr>
<tr>
<td>Perigee Altitude, km</td>
</tr>
<tr>
<td>220</td>
</tr>
<tr>
<td>Apogee Altitude, km</td>
</tr>
<tr>
<td>2100</td>
</tr>
<tr>
<td>Orbit Inclination, deg.</td>
</tr>
<tr>
<td>49</td>
</tr>
<tr>
<td>Active Operation Period, days</td>
</tr>
<tr>
<td>90</td>
</tr>
<tr>
<td>Launch Vehicle</td>
</tr>
<tr>
<td>11K63</td>
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</tbody>
</table>
Radiophysical method

Total frequency shift of GNSS electromagnetic radiation

\[ \Delta f \Sigma = \Delta fD + \Delta fg \]

where \( \Delta f \Sigma \) – total frequency shift
\( \Delta fD \) – Doppler frequency shift
\( \Delta fg \) – gravitational frequency shift

Gravitational shift of signal frequency

\[ \frac{\Delta f_g}{f_0} = \frac{f_0 - f_1}{f_0} = \frac{u_0 - u_1}{c^2} \]  

where \( f \) – frequency
\( u \) – gravitational potential
\( c \) - speed of light

\[ u = \frac{a_{00}}{R} = \frac{\gamma M}{R} \]

\( \gamma M = 3,98 \cdot 10^5 \text{ km}^3/\text{sec}^2 \)

\[ \frac{\Delta f}{f_0} = \frac{\Delta u}{c^2} = \frac{a_{00}}{R_E + L} \frac{1}{R_E} = \frac{\gamma M}{c^2} \left( \frac{1}{R_E + L} - \frac{1}{R_E} \right) = 4.46 \cdot 10^{-10} \]  

This value exceeds much on-board frequency standard of existing GNSS error value of 10^{-13}. The above fact demonstrates principal technical feasibility of radiophysical method of acceleration of gravity value measurement implementation.
Well-known Pound-Rebka ratio is introduced below (awarded by the Nobel Prize in 1926)

\[ \frac{\Delta f}{f_0} = \frac{g\Delta R}{c^2} \]  

(3)

Comparing (2) and (3) we will obtain the following

\[ gR = \frac{\gamma M}{R} \]

This is well-known equation for Newton’s law of gravitation

\[ F = mg = \frac{\gamma m M}{R^2} \]

However, for circular orbital motion under Earth’s gravity one should use the following ratio

\[ \frac{mV^2}{R_0} = mg \]  

(4)

\[ u = V^2 = gR \]

If we substitute ratio (4) into ratio (3) we will obtain the ratio for ground-based measurements

\[ \frac{\Delta f}{f_0} = \frac{u_0 - V_0^2}{c^2} \]  

(5)

\[ u_0 = V_0^2 + \frac{\Delta f}{f_0}c^2 \]  

(6)

\[ V_0^2 + \frac{\Delta f}{f_0}c^2 \]

\[ g = \frac{\Delta f}{H_0} \]  

(7)

Such measurements method will be implemented under STCU Project #3856
Radio physical method of measurement of gravitational field parameters is introduced in three variants:

1. Measurement of frequency gravitational shift on SC board using GNSS signals

\[ \frac{\Delta f_g}{f_0} = \frac{V_0^2 - V_1^2}{c^2} \]  \hspace{1cm} (8)

2. Measurement of frequency gravitational shift on Earth using GNSS signals

\[ \frac{\Delta f_g}{f_0} = \frac{u_0 - V_1^2}{c^2} \]  \hspace{1cm} (9)

3. Measurement of frequency gravitational shift on Earth (Pound-Rebka experiment)

\[ \frac{\Delta f_g}{f_0} = \frac{u_0 - u_1}{c^2} = \frac{g \Delta H}{c^2} \]  \hspace{1cm} (10)
One could calculate relative gravitational shift of solar system planets radiation relative to Earth using formula (8)

\[
\Delta f_g \frac{f}{f_0} = \frac{V_0^2 - V_1^2}{c^2}
\]

<table>
<thead>
<tr>
<th>Planets</th>
<th>Parameters</th>
<th>R (10^6) km</th>
<th>(V_{\text{orb}}) Km/sec</th>
<th>(\Delta f / f_0) (10^{-10})</th>
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</thead>
<tbody>
<tr>
<td>Mercury</td>
<td></td>
<td>58</td>
<td>48.8</td>
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<tr>
<td>Venus</td>
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<td>108</td>
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<tr>
<td>Earth</td>
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<td>29.8</td>
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<td>4.666</td>
<td>96.25</td>
</tr>
</tbody>
</table>
Conclusion

• Technical characteristics of existing GNSS allow to determine gravitational potential and absolute value of acceleration of gravity

• It is necessary to use frequency standards with relative stability not less than 10⁻¹⁵ to determine gravitational field fluctuations at a level of 10 mGal

• Use of the proposed approach will enable additional GNSS functionality that will allow to solve not only geodesic tasks, but also a number of important applied issues, particularly:
  – gravitational minerals survey
  – seismic activity forecast
  – analysis of gravitational anomalies influence at psychophysical state of people