Rule Learning from Knowledge Graphs Guided by Embedding Models

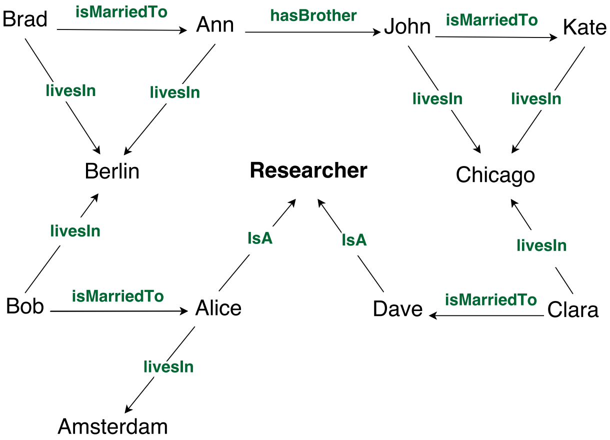
Vinh Thinh Ho[1], Daria Stepanova[1], Mohamed Gad-Elrab[1],
Evgeny Kharlamov[2], Gerhard Weikum[1]

[1]Max Planck Institute for Informatics, Saarbrücken, Germany

[2]University of Oxford, Oxford, United Kingdom

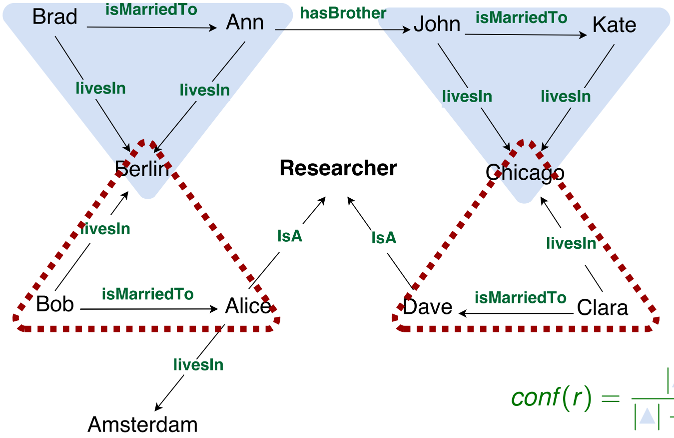
ISWC 2018

Rule Learning from KGs



Rule Learning from KGs

Confidence, e.g., WARMER [Goethals and den Bussche, 2002]
CWA: whatever is missing is false

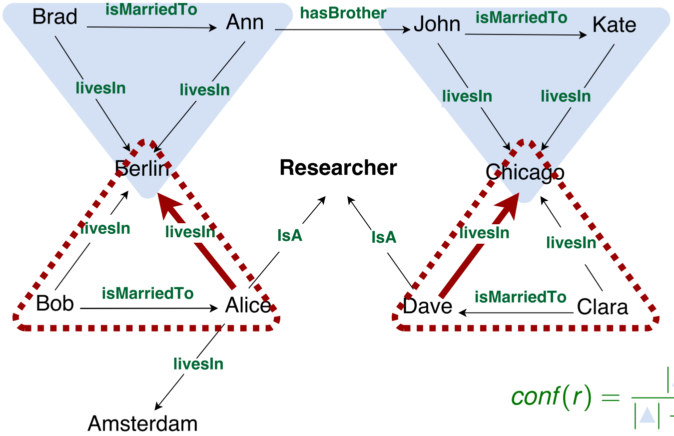


$$conf(r) = \frac{|\blacktriangle|}{|\blacktriangle| + |\triangle|} = \frac{2}{4}$$

$$r : \ livesIn(X, Y) \leftarrow isMarriedTo(Z, X), livesIn(Z, Y)$$

Rule Learning from KGs

Confidence, e.g., WARMER [Goethals and den Bussche, 2002]
CWA: whatever is missing is false



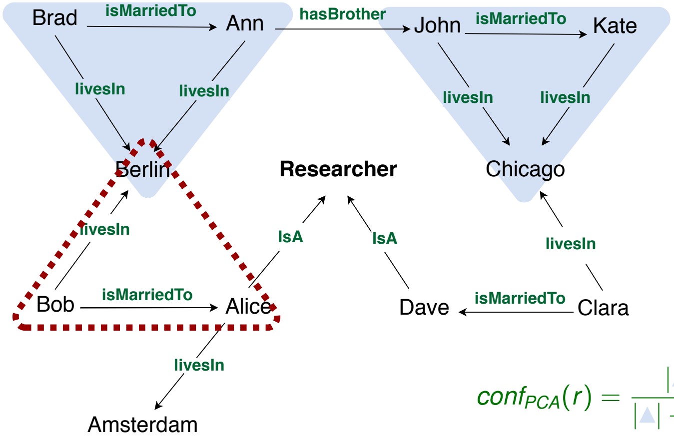
$$conf(r) = \frac{|▲|}{|▲| + |△|} = \frac{2}{4}$$

$r: \ livesIn(X, Y) \leftarrow isMarriedTo(Z, X), livesIn(Z, Y)$

Rule Learning from KGs

PCA confidence AMIE [Galárraga *et al.*, 2015]

PCA: Since Alice has a living place already, all others are incorrect.

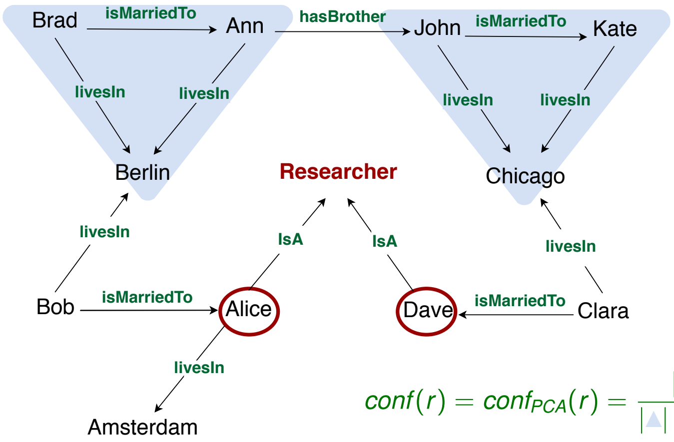


$$conf_{PCA}(r) = \frac{|\blacktriangle|}{|\blacktriangle| + |\triangle|} = \frac{2}{3}$$

$$r : \ livesIn(X, Y) \leftarrow isMarriedTo(Z, X), livesIn(Z, Y)$$

Rule Learning from KGs

Exception-enriched rules: [ISWC 2016, ILP 2016]

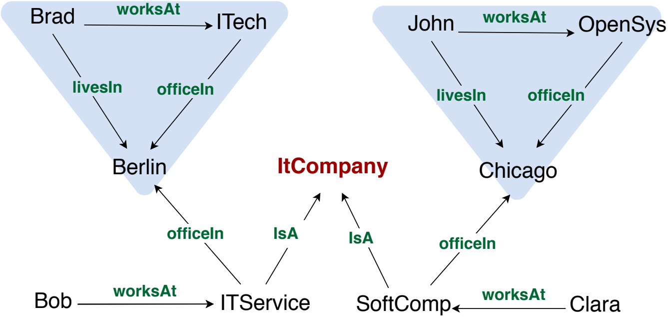


$$conf(r) = conf_{PCA}(r) = \frac{|\text{▲}|}{|\text{▲}| + |\text{♻}|} = 1$$

$$r:\ livesIn(X, Y) \leftarrow isMarriedTo(Z, X), livesIn(Z, Y), not\ isA(X, researcher)$$

Absurd Rules due to Data Incompleteness

Problem: rules learned from highly incomplete KGs might be absurd..

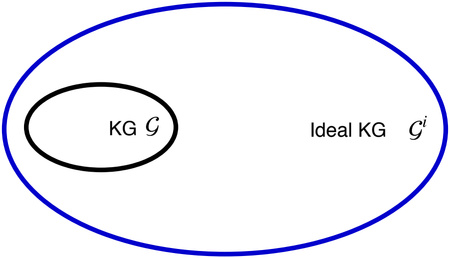


$$conf(r) = conf_{PCA}(r) = 1$$

$$livesIn(X,Y) \leftarrow worksAt(X,Z), officeIn(Z,Y), not\ isA(Z, itCompany)$$

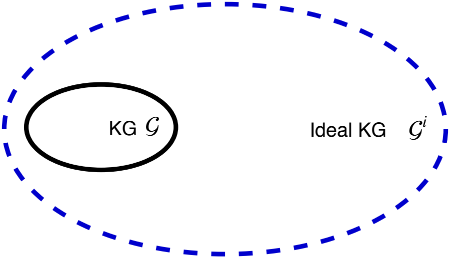
Ideal KG

$\mu(r, \mathcal{G}^i)$: measure quality of the rule r on $\mathcal{G}^i$



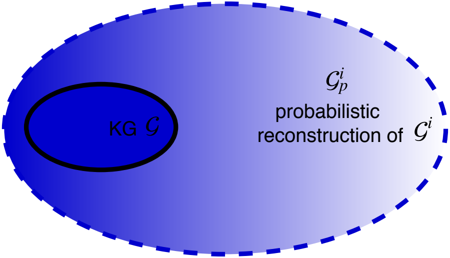
Ideal KG

$\mu(r, \mathcal{G}^i)$: measure quality of the rule r on $\mathcal{G}^i$, but $\mathcal{G}^i$ is unknown



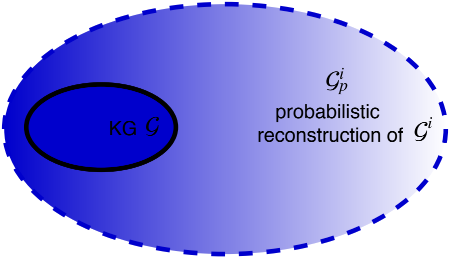
Probabilistic Reconstruction of Ideal KG

$\mu(r, \mathcal{G}_p^i)$: measure quality of r on $\mathcal{G}_p^i$



Hybrid Rule Measure

$$\mu(r, \mathcal{G}_p^i) = (1 - \lambda) \times \mu_1(r, \mathcal{G}) + \lambda \times \mu_2(r, \mathcal{G}_p^i)$$



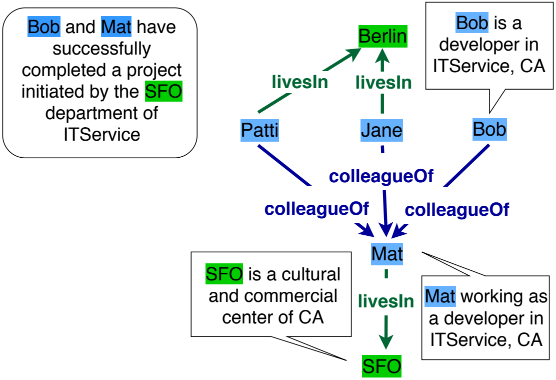
Hybrid Rule Measure

$$\mu(r, \mathcal{G}_p^i) = (1 - \lambda) \times \mu_1(r, \mathcal{G}) + \lambda \times \mu_2(r, \mathcal{G}_p^i)$$

- **$\lambda \in [0..1]$: weighting factor**
- **μ_1 : descriptive quality** of rule r over the available KG $\mathcal{G}$
    - confidence
    - PCA confidence
- **μ_2 : predictive quality** of r relying on $\mathcal{G}_p^i$ (probabilistic reconstruction of the ideal KG $\mathcal{G}^i$)

KG Embeddings

- Popular approach to KG completion, which proved to be effective
- Relies on translation of entities and relations into vector spaces

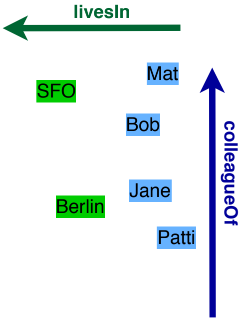


Bob and Mat have successfully completed a project initiated by the SFO department of ITService

Bob is a developer in ITService, CA

SFO is a cultural and commercial center of CA

Mat working as a developer in ITService, CA



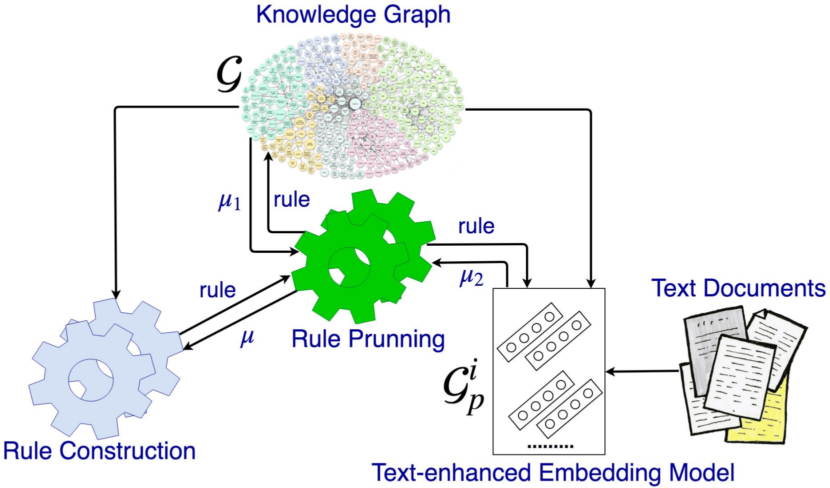
score(<Bob **livesIn** SFO>) = 0.8

score(<Bob **livesIn** Berlin>) = 0.4

...

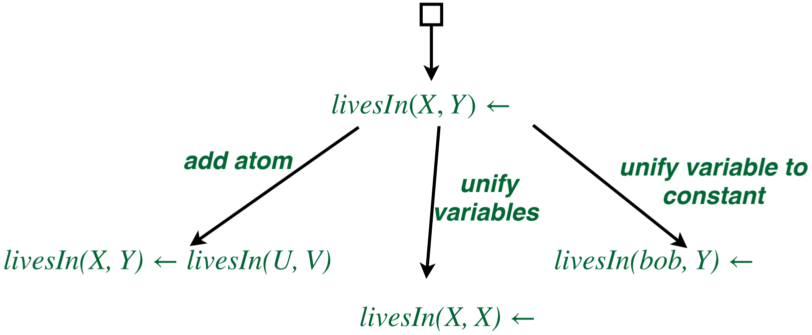
TransE [Bordes *et al.*, 2013], SSP [Xiao *et al.*, 2017], TEKE [Wang and Li, 2016]

Our Approach



Rule Construction

- Clause exploration from general to specific
    - all first-order clauses: [Shapiro, 1991]



$\Box$

$livesIn(X, Y) \leftarrow$

add atom

unify variables

unify variable to constant

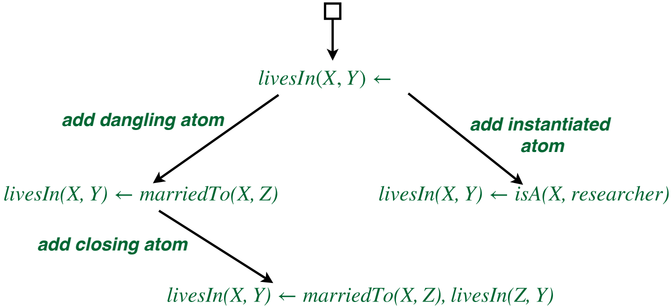
$livesIn(X, Y) \leftarrow livesIn(U, V)$

$livesIn(X, X) \leftarrow$

$livesIn(bob, Y) \leftarrow$

Rule Construction

- Clause exploration from general to specific
  - closed rules: AMIE [Galárraga *et al.*, 2015]
    $\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y)$



$\square$

$\mathit{livesIn}(X, Y) \leftarrow$

add dangling atom

add instantiated atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z)$

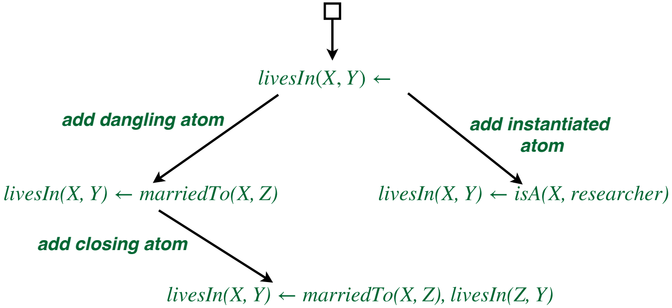
$\mathit{livesIn}(X, Y) \leftarrow \mathit{isA}(X, \mathit{researcher})$

add closing atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y)$

Rule Construction

- Clause exploration from general to specific
  - closed rules: AMIE [Galárraga *et al.*, 2015]
    $livesIn(X, Y) \leftarrow marriedTo(X, Z), livesIn(Z, Y)$



$\square$

$livesIn(X, Y) \leftarrow$

add dangling atom

add instantiated atom

$livesIn(X, Y) \leftarrow marriedTo(X, Z)$

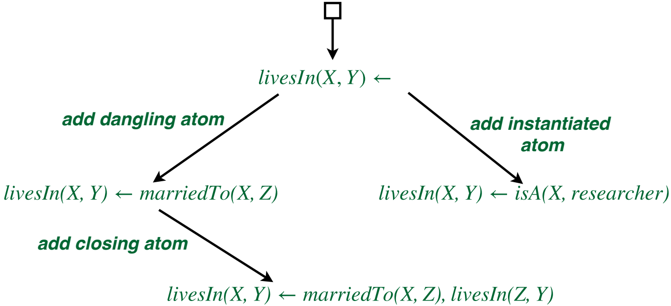
$livesIn(X, Y) \leftarrow isA(X, researcher)$

add closing atom

$livesIn(X, Y) \leftarrow marriedTo(X, Z), livesIn(Z, Y)$

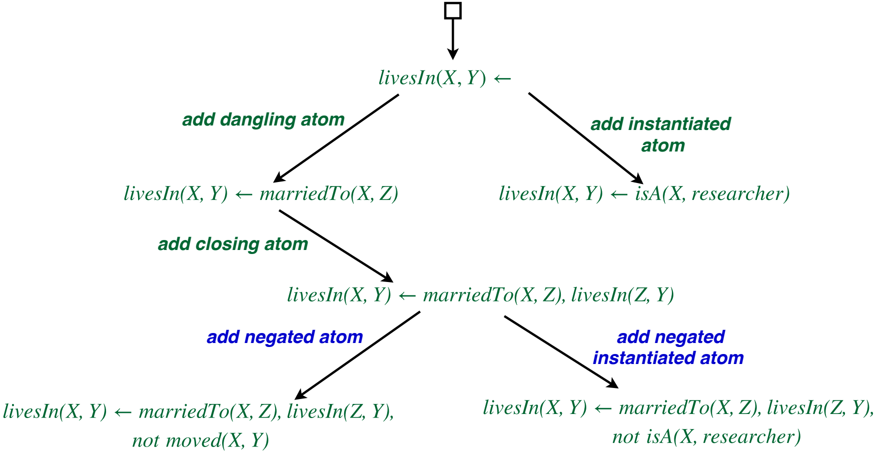
Rule Construction

- Clause exploration from general to specific
  - closed rules: AMIE [Galárraga *et al.*, 2015]
    $livesIn(X, Y) \leftarrow marriedTo(X, Z), livesIn(Z, Y)$



Rule Construction

- Clause exploration from general to specific
  - This work: closed and safe rules with negation
    $\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y), \mathit{not\ isA}(X, \mathit{researcher})$



$\mathit{livesIn}(X, Y) \leftarrow$

add dangling atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z)$

add instantiated atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{isA}(X, \mathit{researcher})$

add closing atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y)$

add negated atom

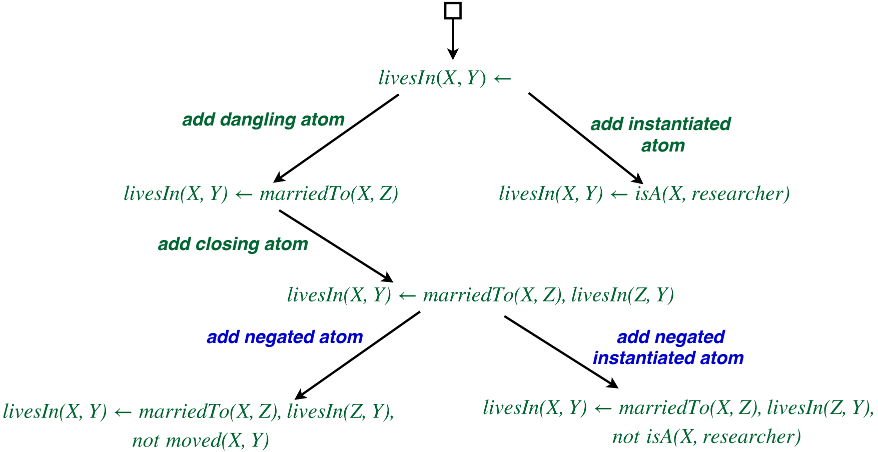
$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y), \mathit{not\ moved}(X, Y)$

add negated instantiated atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y), \mathit{not\ isA}(X, \mathit{researcher})$

Rule Construction

- Clause exploration from general to specific
  - This work: closed and safe rules with negation
    $\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y), \mathit{not}\ \mathit{isA}(X, \mathit{researcher})$



$\Box$

$\mathit{livesIn}(X, Y) \leftarrow$

add dangling atom

add instantiated atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z)$

$\mathit{livesIn}(X, Y) \leftarrow \mathit{isA}(X, \mathit{researcher})$

add closing atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y)$

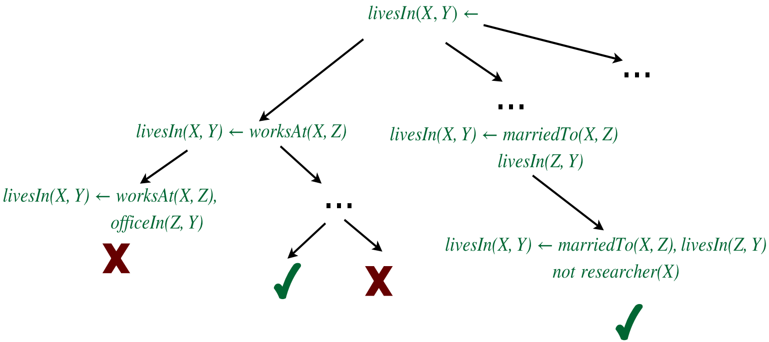
add negated atom

add negated instantiated atom

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y), \mathit{not}\ \mathit{moved}(X, Y)$

$\mathit{livesIn}(X, Y) \leftarrow \mathit{marriedTo}(X, Z), \mathit{livesIn}(Z, Y), \mathit{not}\ \mathit{isA}(X, \mathit{researcher})$

Rule Prunning



livesIn(X, Y) ←

…

…

livesIn(X, Y) ← worksAt(X, Z)

livesIn(X, Y) ← marriedTo(X, Z)
livesIn(Z, Y)

livesIn(X, Y) ← worksAt(X, Z),
officeIn(Z, Y)

✗

…

✓ ✗

livesIn(X, Y) ← marriedTo(X, Z), livesIn(Z, Y)
not researcher(X)

✓

Prune rule search space relying on

- novel hybrid embedding-based rule measure

Embedding-based Rule Quality

- Estimate average quality of predictions made by a given rule r

$$\mu_2(r, \mathcal{G}_p^i) = \frac{1}{|predictions(r, \mathcal{G})|} \sum_{fact \in predictions(r, \mathcal{G})} \mathcal{G}_p^i(fact)$$

  - Rely on truthfulness of **predictions made by** r based on the probabilistic reconstruction $\mathcal{G}_p^i$ of $\mathcal{G}^i$

Embedding-based Rule Quality

- Estimate average quality of predictions made by a given rule *r*

$$\mu_2(r, \mathcal{G}_p^i) = \frac{1}{|predictions(r, \mathcal{G})|} \sum_{fact \in predictions(r, \mathcal{G})} \mathcal{G}_p^i(fact)$$

  - Rely on truthfulness of **predictions made by** *r* based on the probabilistic reconstruction $\mathcal{G}_p^i$ of $\mathcal{G}^i$

Example:

$$livesIn(X, Y) \leftarrow marriedTo(X, Z), livesIn(Z, Y)$$

- Rule predictions: $livesIn(mat, monterey)$,$livesIn(dave, chicago)$

$$\mu_2(r, \mathcal{G}_p^i) = \frac{\mathcal{G}_p^i(<\text{mat } \textbf{livesIn} \text{ monterey}>) + \mathcal{G}_p^i(<\text{dave } \textbf{livesIn} \text{ chicago}>)}{2}$$

Embedding-based Rule Quality

- Estimate average quality of predictions made by a given rule r

$$\mu_2(r, \mathcal{G}_p^i) = \frac{1}{|predictions(r, \mathcal{G})|} \sum_{fact \in predictions(r, \mathcal{G})} \mathcal{G}_p^i(fact)$$

  - Rely on truthfulness of **predictions made by** r based on the probabilistic reconstruction $\mathcal{G}_p^i$ of $\mathcal{G}^i$

Example:

$livesIn(X, Y) \leftarrow marriedTo(X, Z), livesIn(Z, Y), not\ isA(X, surfer)$

- Rule predictions: ~~$livesIn(mat, monterey)$~~, $livesIn(dave, chicago)$

$$\mu_2(r, \mathcal{G}_p^i) = \frac{\mathcal{G}_p^i(<\text{dave } \textbf{livesIn} \text{ chicago}>)}{1}$$

- $\mu_2(r, \mathcal{G}_p^i)$ goes down for noisy exceptions

Evaluation Setup

- Datasets:
    - FB15K: 592K facts, 15K entities and 1345 relations
    - Wiki44K: 250K facts, 44K entities and 100 relations

- Training graph $\mathcal{G}$: remove 20% from the available KG

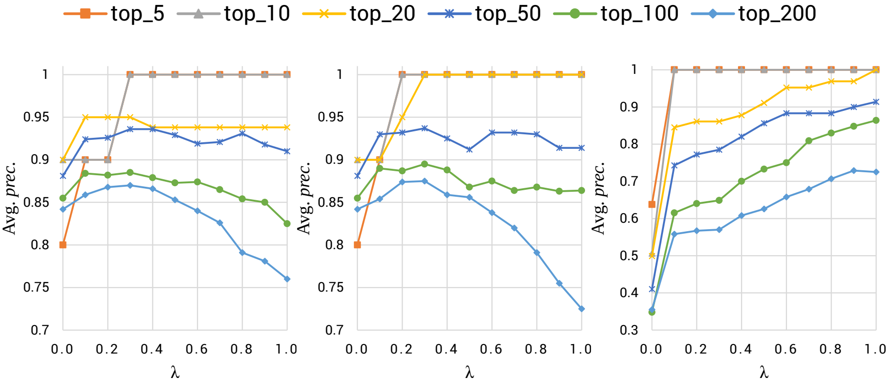
- Embedding models $\mathcal{G}_p^i$:
    - TransE [Bordes *et al.*, 2013], HolE [Nickel *et al.*, 2016]
    - With text: SSP [Xiao *et al.*, 2017]

- Goals:
    - Evaluate effectiveness of our hybrid rule measure

$$\mu(r, \mathcal{G}_p^i) = (1 - \lambda) \times \mu_1(r, \mathcal{G}) + \lambda \times \mu_2(r, \mathcal{G}_p^i)$$

- Compare against state-of-the-art rule learning systems

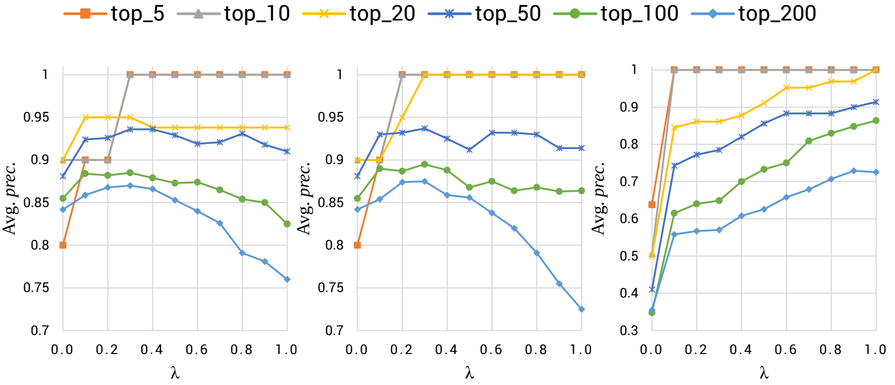
Evaluation of Hybrid Rule Measure



(a) Conf-HolE (b) Conf-SSP (c) PCA-SSP

Precision of *top-k* rules ranked using variations of μ on FB15K.

Evaluation of Hybrid Rule Measure



(a) Conf-HolE (b) Conf-SSP (c) PCA-SSP

Precision of *top-k* rules ranked using variations of μ on FB15K.

- Positive impact of embeddings in all cases for $\lambda = 0.3$
- **Note:** in (c) comparison to AMIE [Galárraga *et al.*, 2015] ($\lambda = 0$)

Example Rules

Examples of rules learned from Wikidata

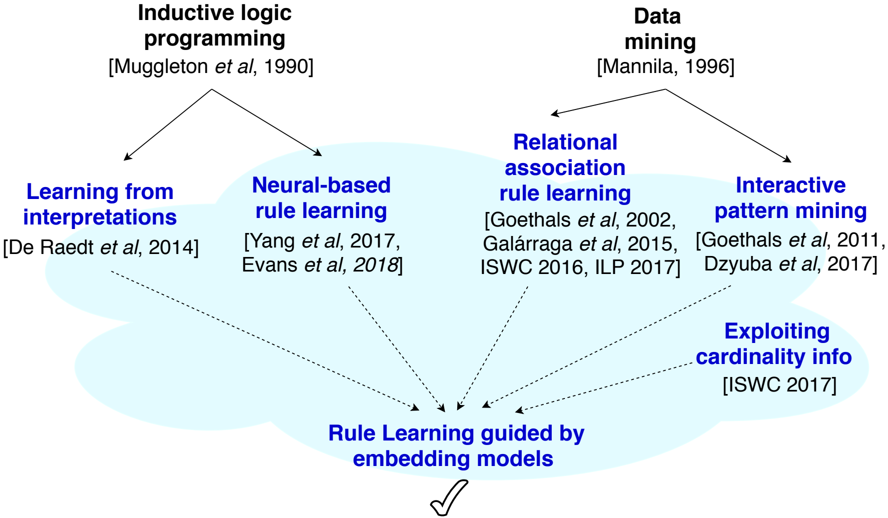
Script writers stay the same throughout a sequel, but not for TV series

$r_1 : \mathit{scriptwriterOf}(X, Y) \leftarrow \mathit{precededBy}(Y, Z), \mathit{scriptwriterOf}(X, Z), \textbf{not } \mathit{isA}(Z, \mathit{tvSeries})$

Nobles are typically married to nobles, but not in the case of Chinese dynasties

$r_2 : \mathit{nobleFamily}(X, Y) \leftarrow \mathit{spouse}(X, Z), \mathit{nobleFamily}(Z, Y), \textbf{not } \mathit{isA}(Y, \mathit{chineseDynasty})$

Related Work



Inductive logic programming
[Muggleton *et al*, 1990]

Data mining
[Mannila, 1996]

Learning from interpretations
[De Raedt *et al*, 2014]

Neural-based rule learning
[Yang *et al*, 2017,
Evans *et al, 2018*]

Relational association rule learning
[Goethals *et al*, 2002,
Galárraga *et al*, 2015,
ISWC 2016, ILP 2017]

Interactive pattern mining
[Goethals *et al*, 2011,
Dzyuba *et al*, 2017]

Exploiting cardinality info
[ISWC 2017]

Rule Learning guided by embedding models

✓

Conclusion

- Summary:
  - Framework for learning rules from KGs with external sources
  - Hybrid embedding-based rule quality measure
  - Experimental evaluation on real-world KGs
  - Approach is orthogonal to a concrete embedding used

- Outlook:
  - Other rule types, e.g., with existentials in the head or constraints
  - Plug-in portfolio of embeddings
  - Mimic framework of exact learning [Angluin, 1987] by establishing complex queries to embeddings

References I

Dana Angluin.
Queries and concept learning.
Machine Learning, 2(4):319–342, 1987.

Antoine Bordes, Nicolas Usunier, Alberto García-Durán, Jason Weston, and Oksana Yakhnenko.
Translating Embeddings for Modeling Multi-relational Data.
In *Proceedings of NIPS*, pages 2787–2795, 2013.

Luis Galárraga, Christina Teflioudi, Katja Hose, and Fabian M. Suchanek.
Fast rule mining in ontological knowledge bases with AMIE+.
VLDB J., 24(6):707–730, 2015.

Bart Goethals and Jan Van den Bussche.
Relational association rules: Getting warmer.
In *PDD*, 2002.

Maximilian Nickel, Lorenzo Rosasco, and Tomaso A. Poggio.
Holographic embeddings of knowledge graphs.
In *AAAI*, 2016.

Ehud Y. Shapiro.
Inductive inference of theories from facts.
In *Computational Logic - Essays in Honor of Alan Robinson*, pages 199–254, 1991.

Zhigang Wang and Juan-Zi Li.
Text-enhanced representation learning for knowledge graph.
In *IJCAI*, 2016.

Han Xiao, Minlie Huang, Lian Meng, and Xiaoyan Zhu.
SSP: semantic space projection for knowledge graph embedding with text descriptions.
In *AAAI*, 2017.