Inference in kinetic Ising models: mean field and Bayes estimators

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NETADIS
Statistical Physics Approaches to Networks Across Disciplines

The kinetic Ising model

- Motivation: study of network reconstruction from dynamical data and reverse engineering of complex biological systems, e.g., gene regulation or neural networks.

Ising spins $s_i = \pm 1, \; i = 1 \dots N$

Synchronous parallel dynamics:

$$P(\sigma_i(t) | \{\sigma_j(t-1)\}_{j=1}^{N}) = \frac{e^{\sigma_i(t) \sum_j J_{ij} \sigma_j(t-1)}}{2\cosh(\sum_j J_{ij} \sigma_j(t-1))},$$

The coupling matrix may be not symmetric and we consider fully connected systems.

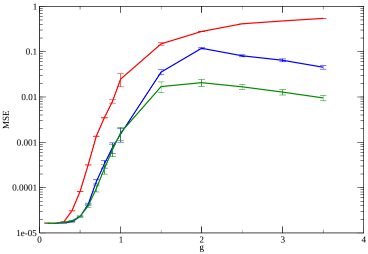
- Temporal sequence of observed spin variables $\rightarrow$ estimate the couplings between sites.

Mean field approach

- Exact inference of the couplings between the sites is not tractable for large networks → approximate inference.
- Mean field theory through an extention of Plefka's (weak coupling) expansion. Effective non interacting description of the dynamics:

$$m_i(t+1) = \left\langle \tanh\left(\sum_j J_{ij} m_j(t) + \Phi_i(t) + \sum_j J_{ij} J_{ji} \sum_{t'}^{t-1} R_i(t,t')(s_i(t') - m_i(t')) \right) \right\rangle_{\Phi_i}$$

$$\langle \Phi_i(t) \Phi_i(t') \rangle = C_i(t,t') m_i(t) m_i(t')$$



Asymmetric networks: inferring the couplings

Network with independently Gaussian distributed random couplings.
Study and compare the theoretical performance of:

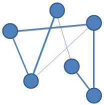
- Mean field predictor (M. Mézard and J. Sakellariou. *J. Stat. Mech: Theor. Exp*, L07001, 2011.)
- Bayes predictor (optimal on average over teacher networks drawn at random from the prior)

  $\Uparrow$

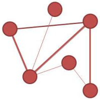
  implementation based on an algorithm of the expectation-propagation (EP) type.

At what rate the error decreases with growing length of trajectories?

Prediction error



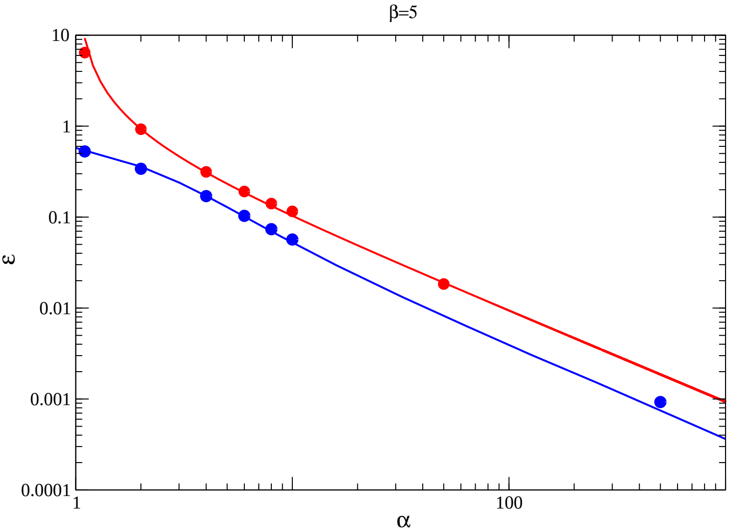
J^*_{ij} 'teacher' network from which the data are generated



J_{ij}: estimated network

$$\varepsilon = N^{-1}\overline{||\mathbf{J}^* - \mathbf{J}||^2}$$

Averages are over the spin trajectories generated by $\mathbf{J}^*$ and over $\mathbf{J}^*$ (replica trick)



L. Bachschmid-Romano and M. Opper. *J.* Stat. Mech., 2015.9 (2015): P09016.