Inference in kinetic Ising models: mean field and Bayes estimators

Ludovica Bachschmid-Romano, Manfred Opper

Artificial Intelligence group, Computer Science, TU Berlin, Germany

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The kinetic Ising model

Motivation: study of network reconstruction from dynamical data and reverse engineering of complex biological systems, e.g., gene regulation or neural networks.

Ising spins $s_i = \pm 1, \ i = 1 \ldots N$

Synchronous parallel dynamics:

$$P(\sigma_i(t)|\{\sigma_j(t-1)\}_{j=1}^{N}) = \frac{e^{\sigma_i(t) \sum_j J_{ij} \sigma_j(t-1)}}{2 \cosh(\sum_j J_{ij} \sigma_j(t-1))},$$

The coupling matrix may be not symmetric and we consider fully connected systems.

Temporal sequence of observed spin variables $\rightarrow$ estimate the couplings between sites.
Mean field approach

- Exact inference of the couplings between the sites is not tractable for large networks $\rightarrow$ approximate inference.

- Mean field theory through an extension of Plefka’s (weak coupling) expansion. Effective non interacting description of the dynamics:

$$m_i(t+1) = \left\langle \tanh \left( \sum_j J_{ij} m_j(t) + \Phi_i(t) + \sum_j J_{ij} J_{ji} \sum_{t'} R_i(t, t')(s_i(t') - m_i(t')) \right) \right\rangle$$

$$\langle \Phi_i(t) \Phi_i(t') \rangle = C_i(t, t') m_i(t) m_i(t')$$
Network with independently Gaussian distributed random couplings. Study and compare the theoretical performance of:


- Bayes predictor (optimal on average over teacher networks drawn at random from the prior) implementation based on an algorithm of the expectation-propagation (EP) type.

At what rate the error decreases with growing length of trajectories?
Prediction error

$J_{ij}^*$: 'teacher' network from which the data are generated

$J_{ij}$: estimated network

$\varepsilon = N^{-1}||J^* - J||^2$

Averages are over the spin trajectories generated by $J^*$ and over $J^*$ (replica trick)
β = 5

\[ \alpha \]

\[ \varepsilon \]